SURFACE MOTION ESTIMATION VIA CONFORMAL MAPS

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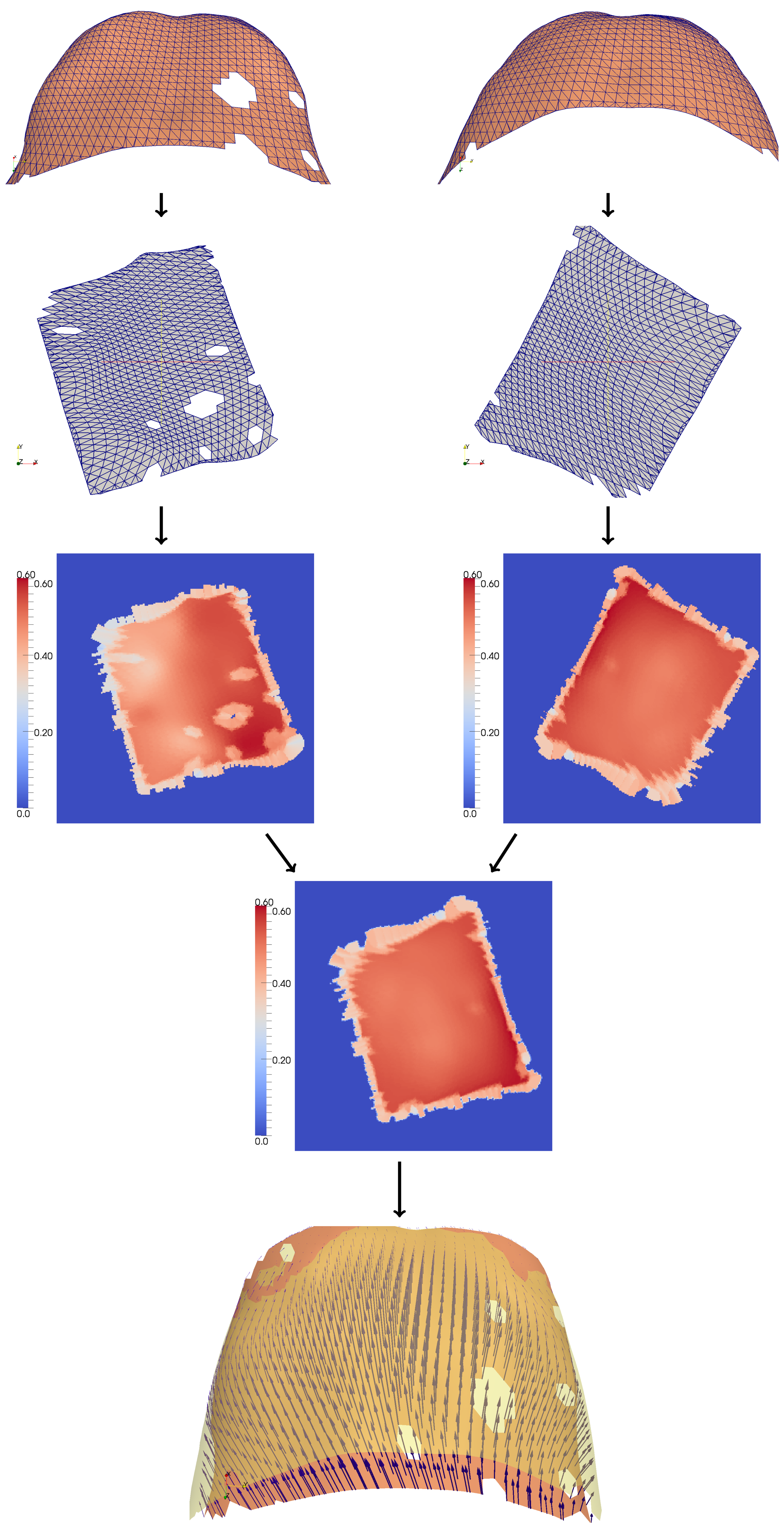
Abstract
Breathing motion challenges precision of radiation therapy on thorax and abdominal regions. Direct measurement is often infeasible and only available through surrogates. This study aims the accurate estimation of surface motion. The proposed method maps 3D surfaces into common 2D parametric plane. Point correspondences are resolved as a registration problem between feature images. The approach will be validated on patient data with skin marks as ground-truth.



References:

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[3] Jamie McClelland. Estimating internal respiratory motion from respiratory surrogate signals using correspondence models. In *4D Modeling and Estimation of Respiratory Motion for Radiation Therapy*, pages 187–213. Springer, 2013.

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Formulation
Conformal maps are angle preserving transformations, used to flatten surfaces. Conformal mappings allows stretching and contraction of lengths while preserving angles.
The map calculation is formulated as the minimization of the conformal energy, a difference between the norm of the map and its area. It can be formulated in a differential geometry framework [1], or equivalently using exterior calculus on k-forms [2].

Input surfaces:
$\mathbf{x}_i = [x_i, y_i, z_i]^T$ position of i-th vertex of surface $\mathcal{X} \subset \mathcal{R}^3$.
e_{ij} edge between i-th and j-th vertex.

Conformal maps:
$\mathbf{u}_i = [u_i, v_i]^T$ position of i-th vertex of plane $\mathcal{U} \subset \mathcal{R}^2$.
$\mathbf{u} = [u_1, v_1, \ldots, u_V, v_V]^T$ for V number of nodes. A vector of $2V$ elements.

$$E_D(\mathbf{u}) = \sum_{e_{ij}} \frac{1}{4}\left(\cot(\theta_{ij}) + \cot(\theta_{ji})\right)(u_i - u_j)^2,$$

where θ_{ij} and θ_{ji} are the angles at the vertex opposite to edge $e_{i,j}$ in $\mathcal{X}$. In matrix form $E_D(\mathbf{u}) = \frac{1}{2}\mathbf{u}^T L_D \mathbf{u}$, L_D a $2V \times 2V$ sparse matrix.
the area is the sum over the boundary vertex

$$\mathcal{A}(\mathbf{u}) = \sum_{e_{i,j} \in \partial\mathcal{U}} \frac{1}{2}(u_i v_j - u_j v_i)$$

in matrix form $\mathcal{A}(\mathbf{u}) = \frac{1}{2}\mathbf{u}^T A \mathbf{u}$, A a $2V \times 2V$ extremely sparse matrix.
The conformal energy $E_C = E_D - A$ is the quadratic form

$$E_C(\mathbf{u}) = \mathbf{u}^T L_C \mathbf{u}, \qquad L_C = L_D - A$$

L_C sparse symmetric matrix.
The solution is then the minimization problem

$$\mathbf{u}^* = \arg\min_{\mathbf{u}} \mathbf{u}^T L_C \mathbf{u}$$

Subject to constraints

- $\mathbf{u}^T \mathbf{e} = 0$. Centered solution on $\mathcal{U}$. $\mathbf{e}$ is a matrix whose columns span the kernel of L_C.
- $\mathbf{u}^T \mathbf{u} = 1$. unit moment of inertia

Solution is boundary free and evenly spread on $\mathcal{U}$. [1]

Feature images:
Interpolation from unstructured triangle mesh on U to a rectangular uniform image grid. With feature value computed from the magnitude of change of area induced by the map. (First form of the surface).

Matching:
Rigid 2D transform (translation and rotation) by image registration.

Motion Field:
Difference between corresponding points $\mathbf{x}_i^{(t_1)}$ and $\mathbf{x}_i^{*(t_2)}$ from surfaces at times t_1 to t_2:

$$\mathbf{d}_i = \mathbf{x}_i^{(t_2)} - \mathbf{x}_i^{(t_1)}$$

Conclusions and Perspectives

- Preliminary results are promising, vector field is smooth and by visual inspection preserves semantic correspondences.
- Hypothesis that breathing motion is mostly dilation contraction and conformal seams to give good estimation of the motion.
- Constraints of the spectral conformal map approach (centered and unit moment of inertia) leave us mostly with residual rotation to be corrected for matching in the parametric plane.
- Results are to be quantified according to ground truth data derived from surface landmarks.
- This study is part of a project whose objective is to develop a correspondece model to predict internal motion from external surface motion as surrogate [3].

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