ROUGH SETS FOR BIAS FIELD CORRECTION IN MR IMAGES USING CONTRAHARMONIC MEAN AND QUANTITATIVE INDEX

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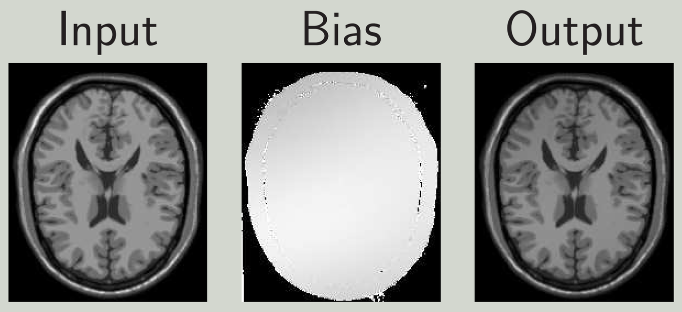
Abstract

The paper presents a novel approach for bias field correction in MR images by judiciously integrating the merits of rough sets and contraharmonic mean. A theoretical analysis is presented to justify the use of both rough sets and contraharmonic mean for bias field estimation. Some new quantitative indices are also introduced to measure the performance of the proposed approach, along with other related approaches, on both simulated and real MR images for different bias fields and noise levels.

Intensity Inhomogeneity



- Causes a shading effect over the image
- Reduces mean and increases variance for all the tissue classes
- Hardly noticeable to a human observer

Bias Correction using HUM

- If i^{th} pixel of the bias-free image, bias field and noise are u_i, b_i and n_i, then i^{th} pixel of the acquired image is: $v_i = u_i b_i + n_i$
- The model of the HUM can be rewritten as

$$u_i = \frac{v_i}{b_i} = \frac{v_i C_N}{LPF(v_i)},$$

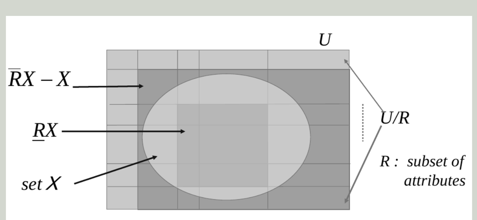
  where $LPF(.)$ is the low-pass filter and C_N is the normalizing constant
- Generally, AM filter is used as mean filter in the HUM

Disadvantages of HUM

- If fixed amount of bias field is applied to two different pixels, then the pixel with higher intensity value should suffer the effect of bias field much more than the other
- Not all the pixels in the neighborhood area of a specific pixel contribute in estimating the bias field

Basics of Rough Sets

- $\underline{R}(X)$ and $\overline{R}(X)$ are called the lower and upper approximations of X
- $B(X) = \overline{R}(X) \setminus \underline{R}(X)$ is the boundary region of X



Contraharmonic Mean for Bias Field Estimation

- The pixel with higher intensity value is given more priority than the others while estimating the bias field
- The CHM filter of order p is defined as follows:

$$\hat{f}_i = \frac{\sum_{j \in N_i} v_j^{p+1}}{\sum_{j \in N_i} v_j^p}$$

  where N_i is the set of pixels in a window centered at i
- Hence, the estimated bias field at coordinate i of the acquired image v is given by

$$b_i'' = \left\{\frac{\sum_{j \in N_i} v_j^{p+1}}{\sum_{j \in N_i} v_j^p}\right\} \left\{\frac{\sum_{j \in I} v_j^{p+1}}{\sum_{j \in I} v_j^p}\right\}^{-1}$$

  where I denotes the set of all pixels in the image

Rough Sets for Bias Field Estimation

- The pixels with similar intensity value contribute more than the other pixels in the neighborhood
- Given an filter N_i of size $\Delta_x \times \Delta_y$ corresponding to the $i = (i_x, i_y)$th pixel, define:

$$\overline{R}(N_i) = \left\{j : \ |j_x - i_x| < \frac{\Delta_x}{2}, |j_y - i_y| < \frac{\Delta_y}{2}\right\};$$

$$\underline{R}(N_i) = \{j : |v_j - v_i| < \delta_i, j \in \overline{R}(N_i)\};$$

$$\text{and } \ B(N_i) = \{j : \ |v_j - v_i| \geq \delta_i, j \in \overline{R}(N_i)\}$$

  where δ_i is a threshold corresponding to the ith pixel.

Rough Set based Bias Field Estimate

The estimated bias field at coordinate i of the acquired image v is

$$b_i''' = \{\omega_i \mathcal{A}_i + (1 - \omega_i)\mathcal{B}_i\} \left\{\frac{\sum_{j \in I} v_j^p}{\sum_{j \in I} v_j^{p+1}}\right\}$$

$$\text{where } \mathcal{A}_i = \frac{\sum_{j \in \underline{R}(N_i)} v_j^{p+1}}{\sum_{j \in \underline{R}(N_i)} v_j^p}; \text{ and } \mathcal{B}_i = \frac{\sum_{j \in B(N_i)} v_j^{p+1}}{\sum_{j \in B(N_i)} v_j^p}$$

ω_i and $(1 - \omega_i)$ represent the relative importance of lower approximation and boundary region of filter N_i

Importance of CHM and Rough Sets

- The CHM filter of order $p > 0$ provides better restoration of MR images than that of the AM filter if the condition $u_i'' < 2u_i - u_i'$ is satisfied, where u_i' and u_i'' are respectively the intensity of the pixel restored by the HUM using AM and CHM filter. [For proof, see [2]]
- Better restoration will be achieved by the rough set based bias field estimation method if

$$\omega_i = \begin{cases} \omega_{i_0} + \epsilon_i & \text{if } \mathcal{A}_i < \mathcal{B}_i \\ \omega_{i_0} - \epsilon_i & \text{if } \mathcal{A}_i > \mathcal{B}_i \\ \omega_{i_0} & \text{if } \mathcal{A}_i = \mathcal{B}_i \end{cases}$$

  where ω_{i_0} is the original weight assigned to the lower approximation region and $\epsilon_i (> 0)$ is constant. [For proof, see [2]]

Proposed Quantitative Measures

- **Index of Class Separability**

$$\text{IoCS} = \min_i \left\{\min_{j \neq i} \left\{\frac{|\mu(\beta_i) - \mu(\beta_j)|}{\max_k \sigma(\beta_k)}\right\}\right\}$$

- **Index of Variation:**

$$\text{IoV} = \frac{1}{c}\sum_{i=1}^{c}\left[1 + \left|1 - \frac{\text{CoV}_{\text{GT}}(\beta_i)}{\text{CoV}_{\text{R}}(\beta_i)}\right|\right]^{-1}$$

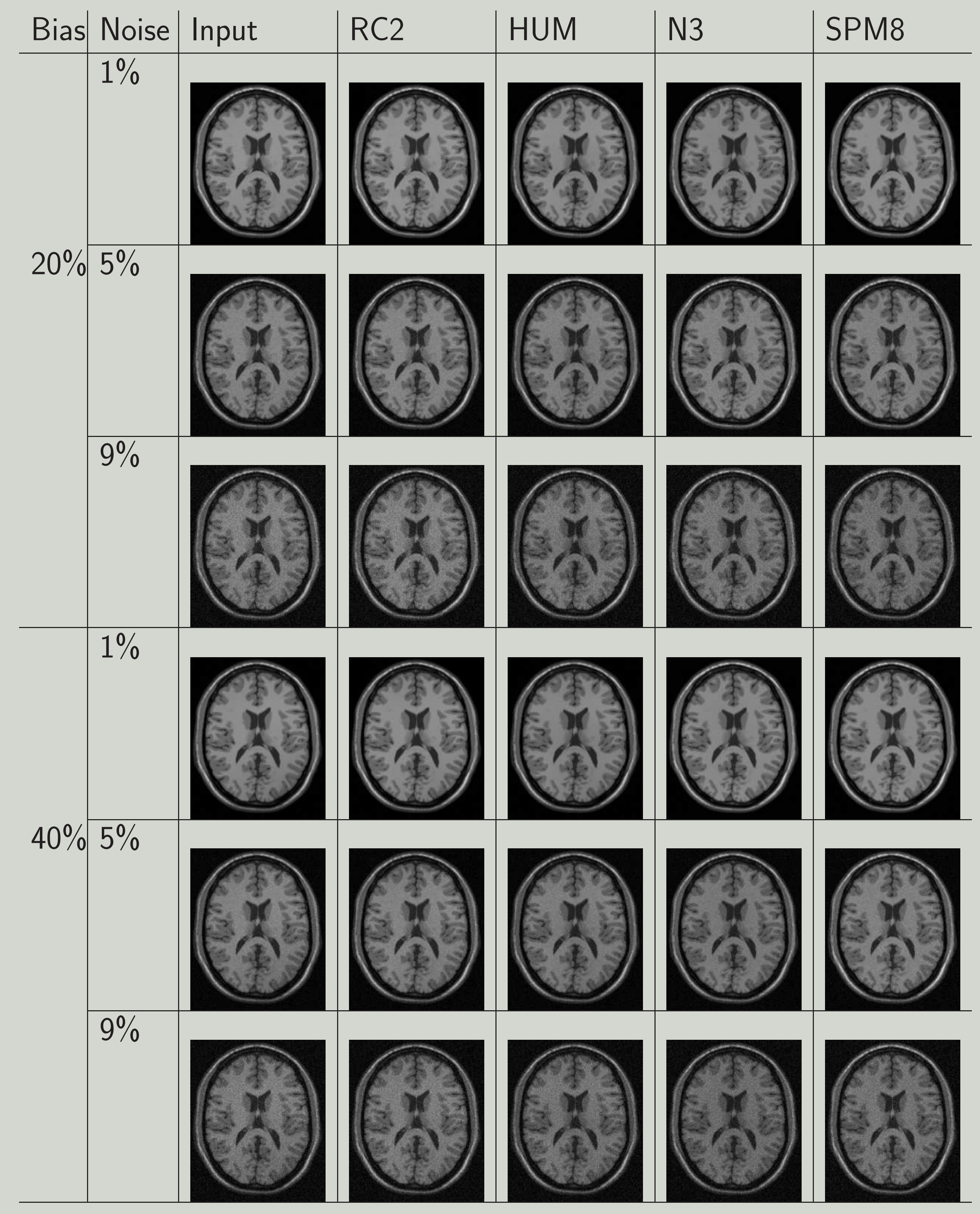
- **Index of Joint Variation**

$$\text{IoJV} = \frac{1}{c}\sum_{i=1}^{c}\max_{j \neq i}\{\text{CoJV}(\beta_i, \beta_j)\}$$

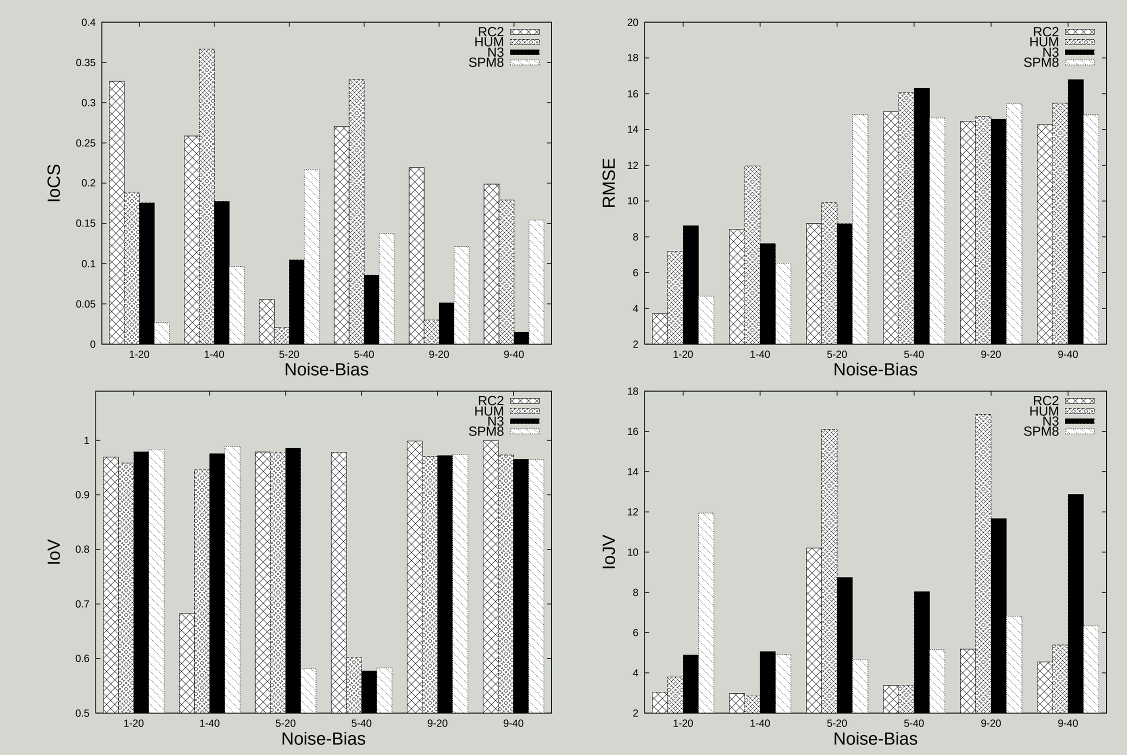
Experiments

- Proposed algorithm (RC2) [2] is compared with the AM based HUM [3], N3 [4] algorithm, and SPM8 software version 8 [1]
- Simulated images are obtained from "BrainWeb: Simulated Brain Database" and real MR images from "IBSR: Internet Brain Segmentation Repository"

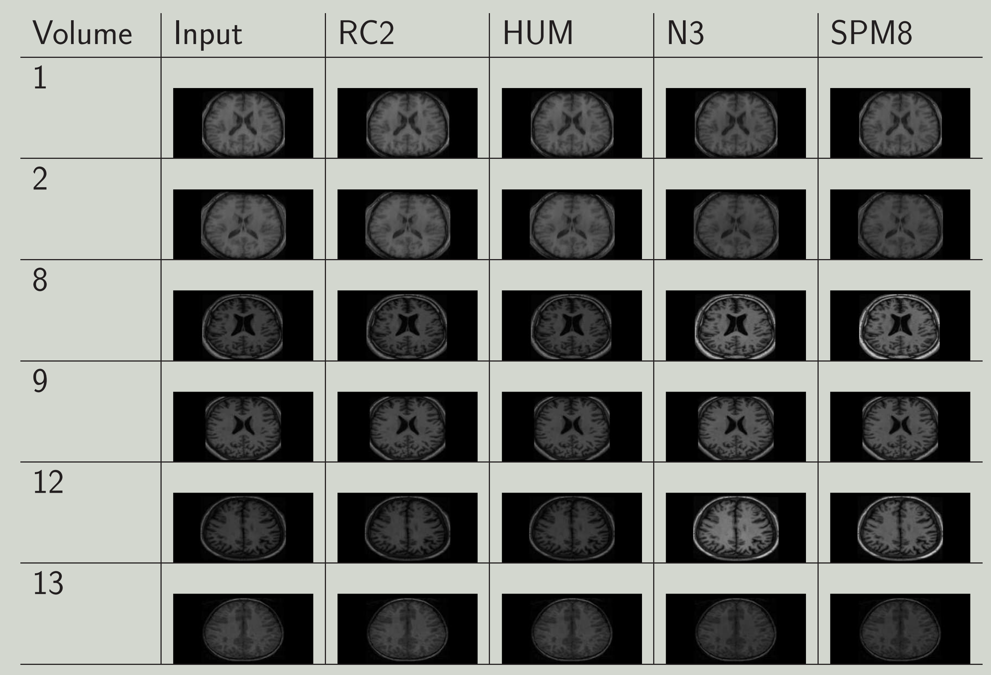
Results: Brainweb



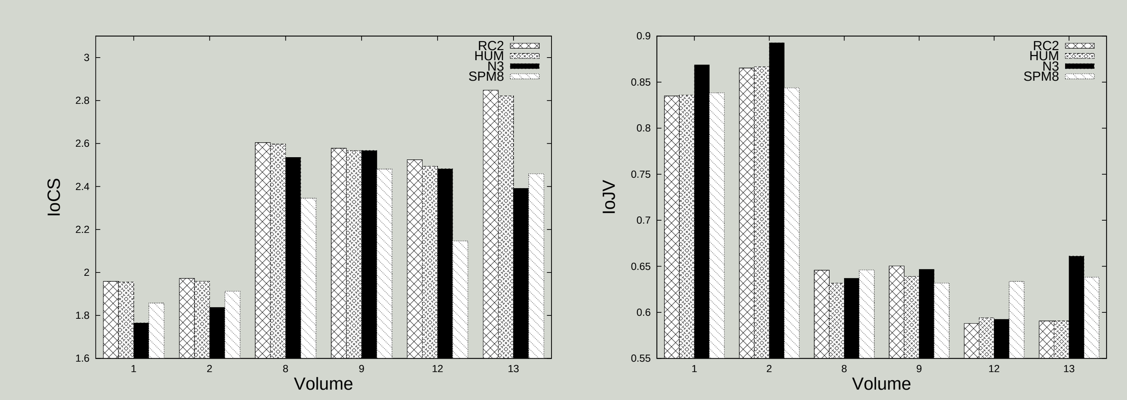
Performance Evaluation: Brainweb



Results: IBSR



Performance Evaluation: IBSR



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