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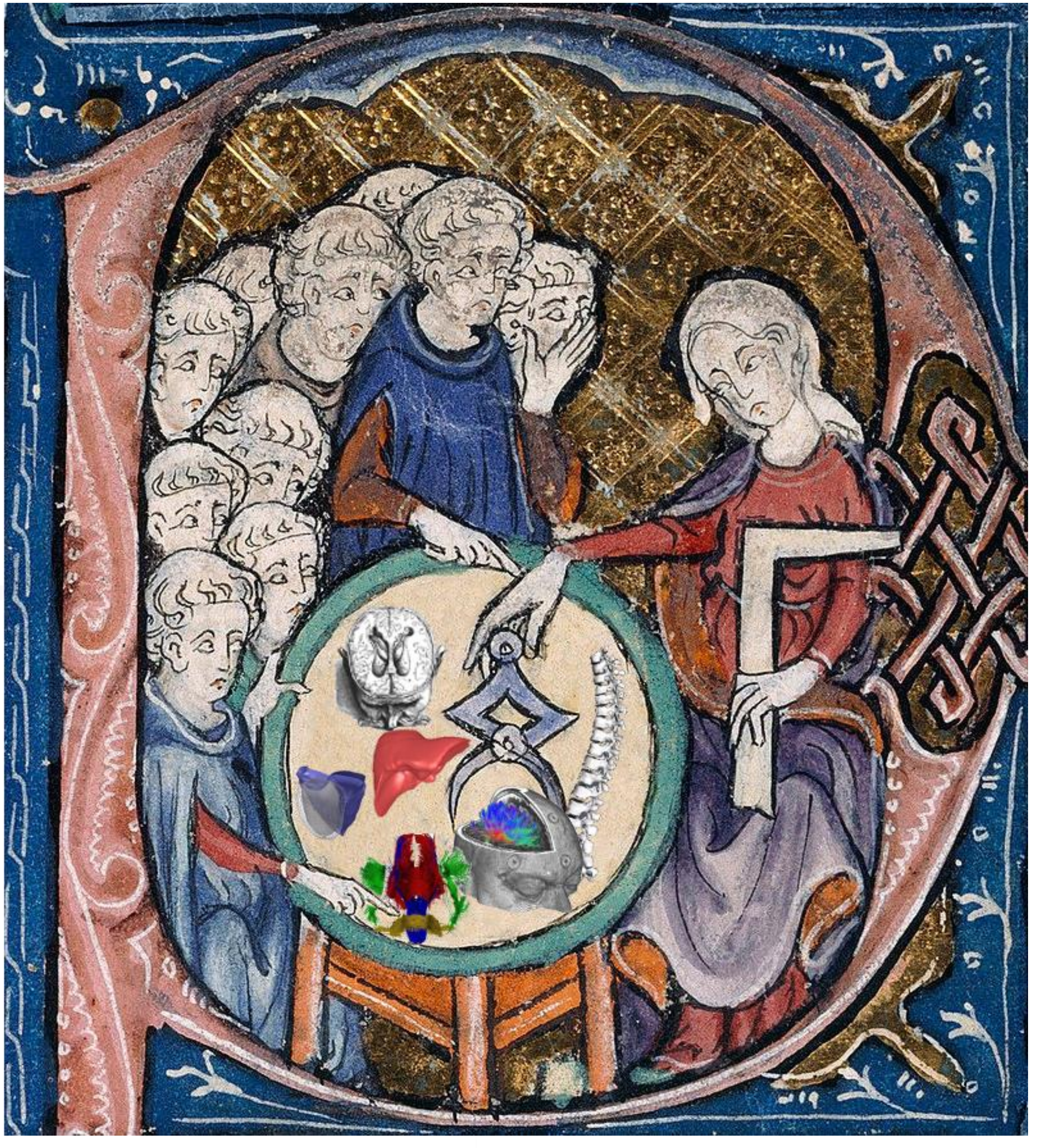
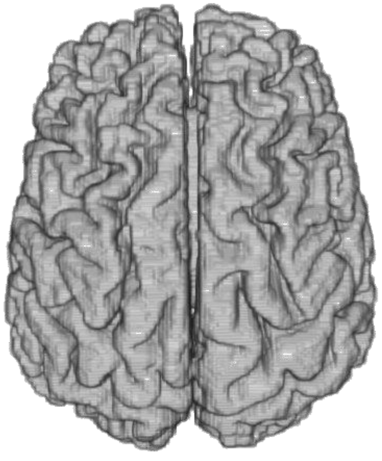
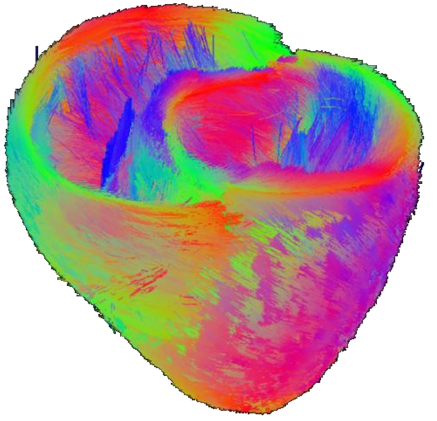


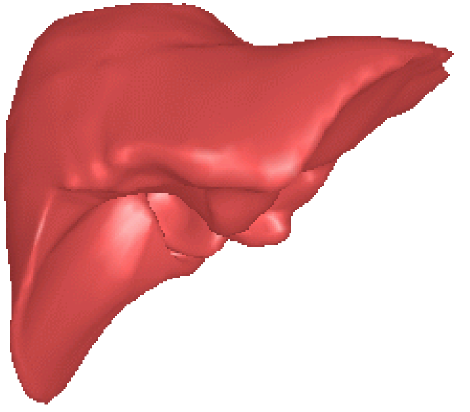
Geometric Structures for Statistics on Shapes and Deformations in Computational Anatomy

MISS Summer School,
Favignana, Sicily, It
July 18-Aug.1 2014

Computational Anatomy



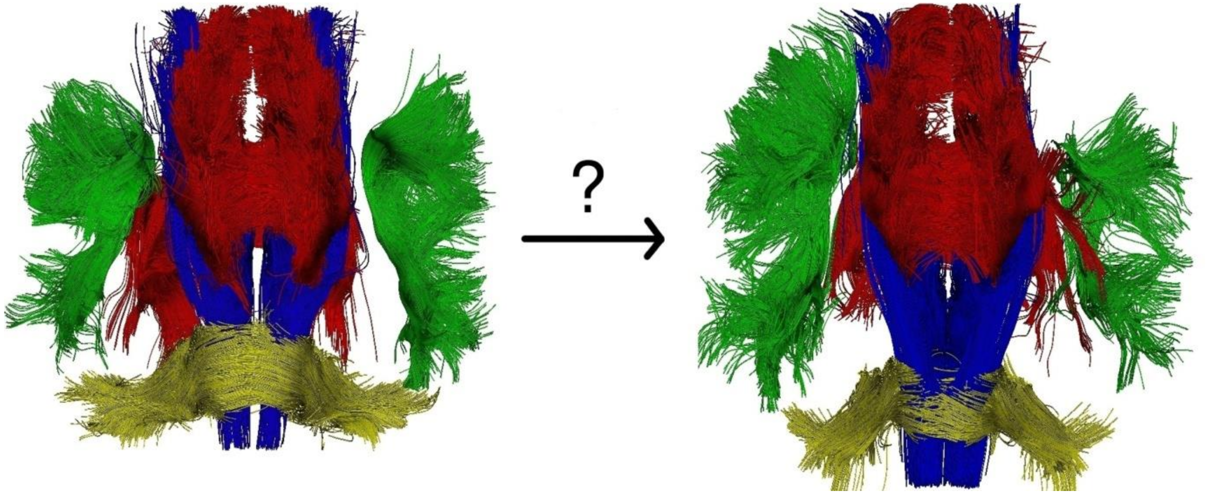




Design Mathematical Methods and Algorithms to Model and Analyze the Anatomy

- Statistics of organ shapes across subjects in species, populations, diseases…
  - Mean shape
  - Shape variability (Covariance)
- Model organ development across time (heart-beat, growth, ageing, ages…)
  - Predictive (vs descriptive) models of evolution
  - Correlation with clinical variables

Shapes: forms & deformations

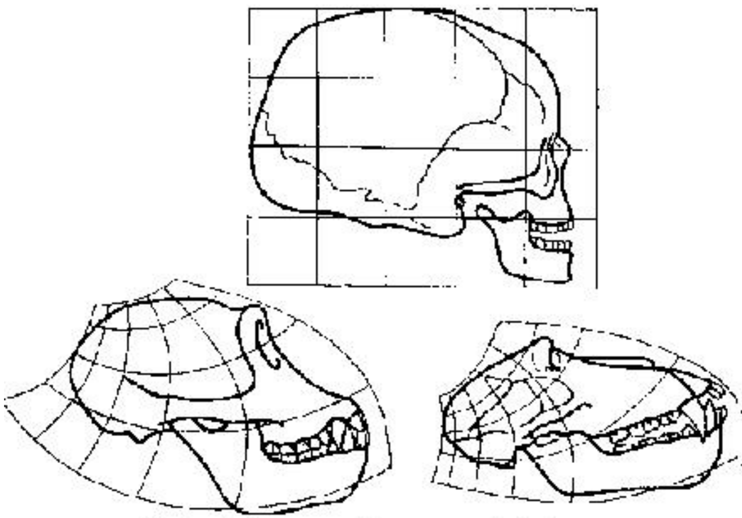


"Shape space" embedding [Kendall]

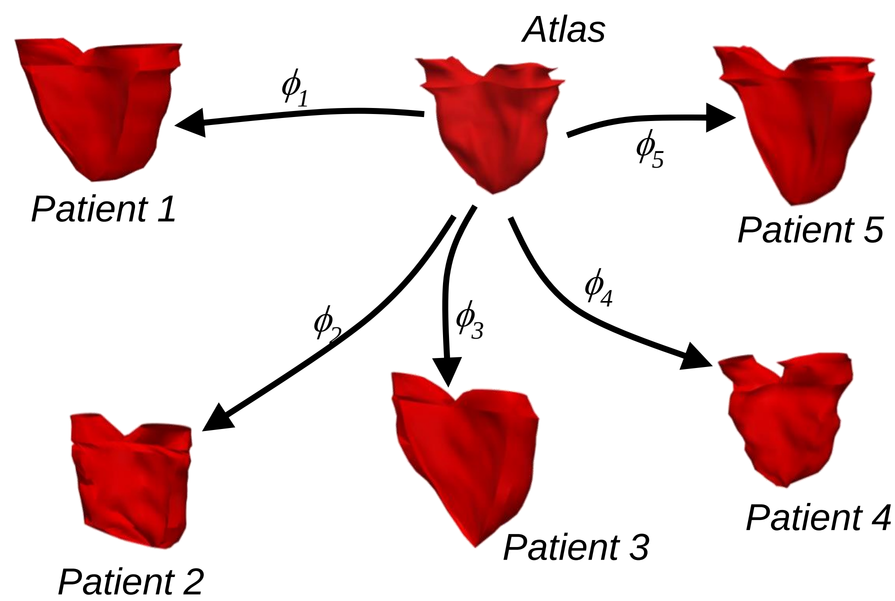
- Shape = what remains from the object when we remove all transformations from a given group
  - Transformation (rigid, similarity, affine) = nuisance factor
  - Shape manifold = quotient of the object manifold by the group action
- Quotient spaces are non-linear (e.g. R^n / scaling = S^n)
- Kendall size & shape space: $(R^n)^d / SO_n$

Statistics on these non-linear spaces?

Morphometry through Deformations



Skulls of a human, a chimpanzee and a baboon and transformations between them



Measure of deformation [D'Arcy Thompson 1917, Grenander & Miller]

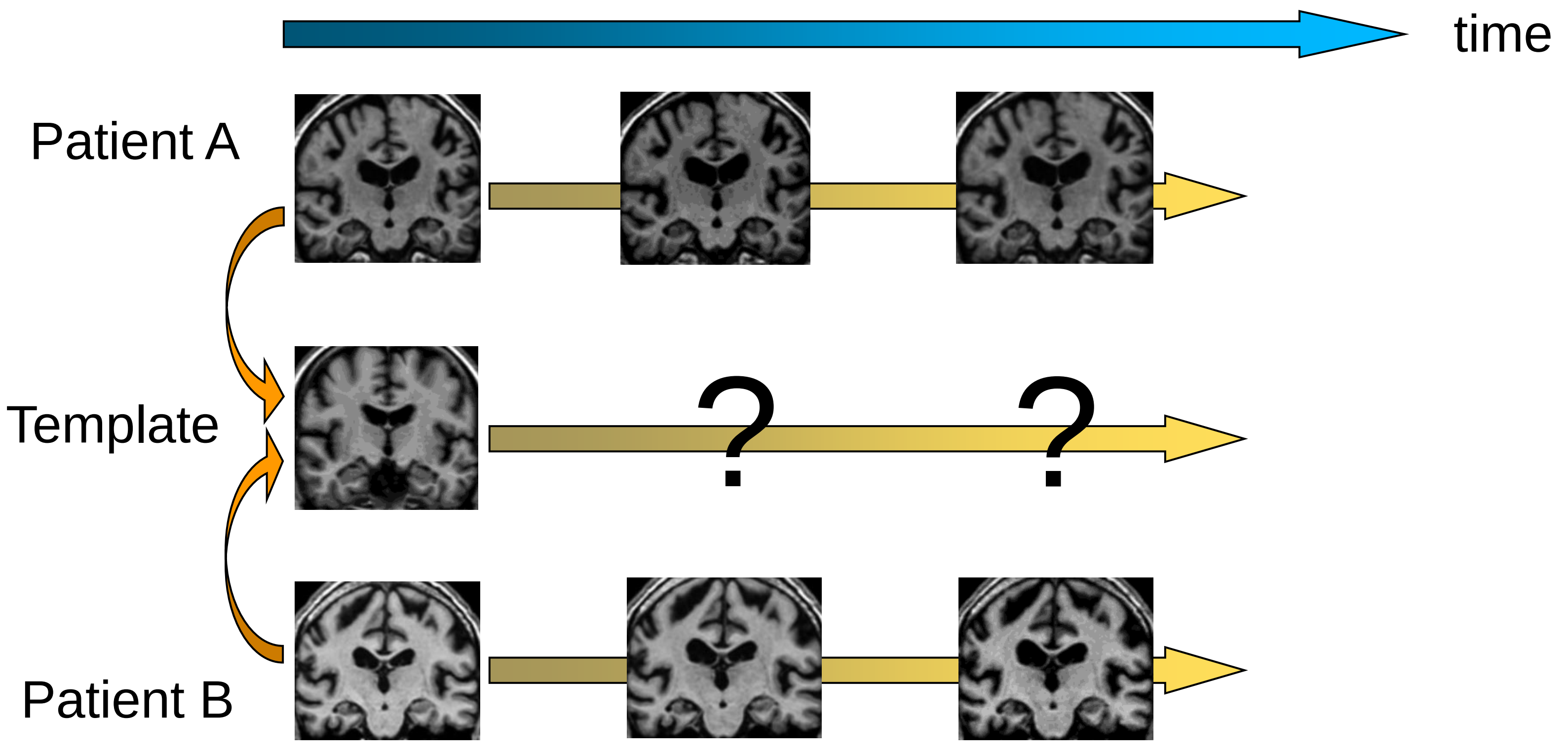
- Observation = "random" deformation of a reference template
- Reference template = Mean (atlas)
- Shape variability encoded by the deformations

Statistics on groups of transformations (Lie groups, diffeomorphism)?

Consistency with group operations (non commutative)?

Longitudinal deformation analysis

Dynamic obervations



How to transport longitudinal deformation across subjects?

What are the convenient mathematical settings?

Outline

Riemannian frameworks on Lie groups

- Manifolds
- Statistics
- Applications to spine shape & heart remodeling

Lie groups as affine connection spaces

- The bi-invariant affine Cartan connection structure
- Extending statistics without a metric

The SVF framework for diffeomorphisms

- Diffeomorphisms with SVFs
- Longitudinal modeling of brain atrophy in AD

Differentiable manifolds

Définition:

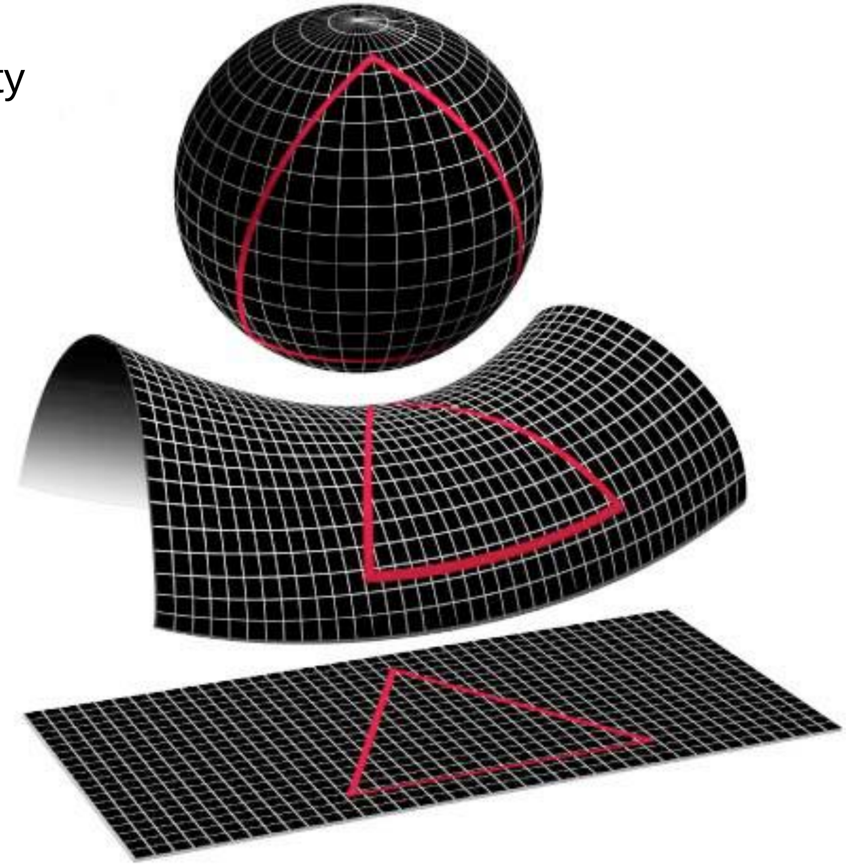
- Locally Euclidean Topological space which can be globally curved
  - Same dimension + differential regularity

Simple Examples

- Sphere
- Saddle (hyperbolic space)
- Surface in 3D space

And less simple ones

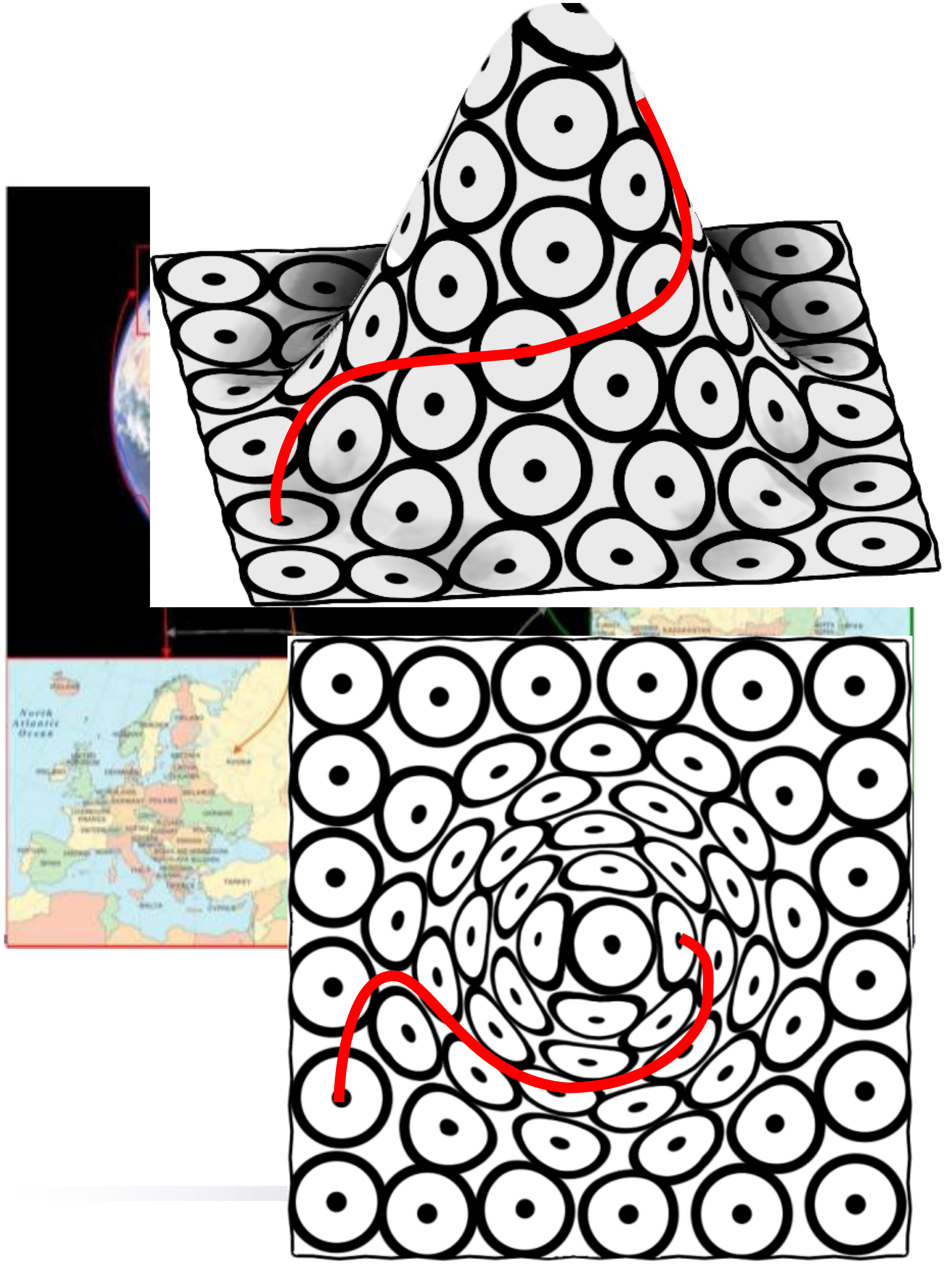
- Projective spaces
- Rotations of R^3 : $SO_3 \sim P_3$
- Rigid, affine Transformation
- Diffeomorphisms

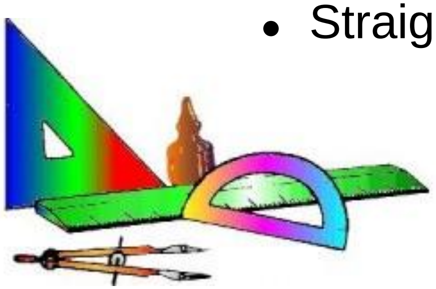


Differentiable manifolds

Computing in a a manifold

- Extrinsic
  - Embedding in $\mathbb{R}^n$
- Intrinsic
  - Coordinates : charts
  - Atlas = consistent set of charts
- Measuring?
  - Volumes (surfaces)
  - Lengths
  - Straight lines





Measuring extrinsic distances

Basic tool: the scalar product

$$< v, w > = v^t w$$

- Norm of a vector

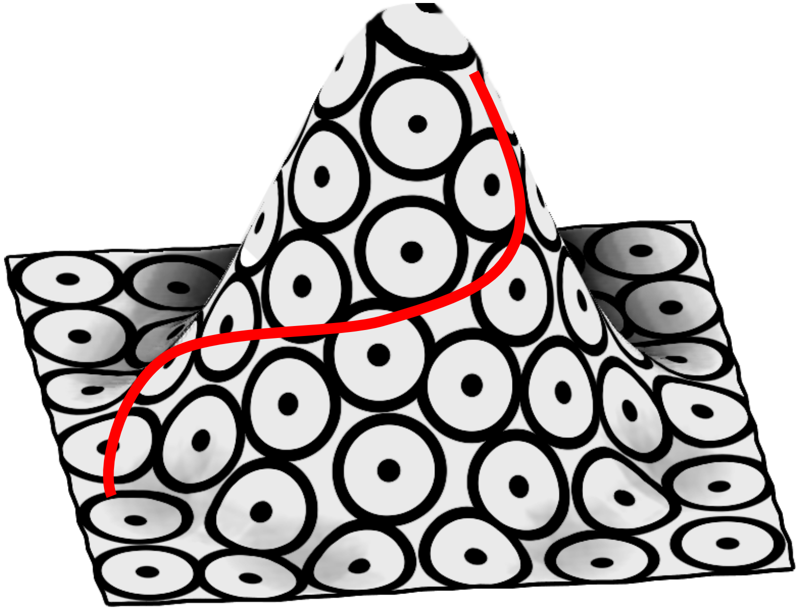
$$\|v\| = \sqrt{< v, v >}$$

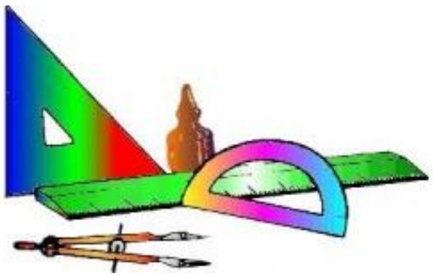
- Angle between vectors

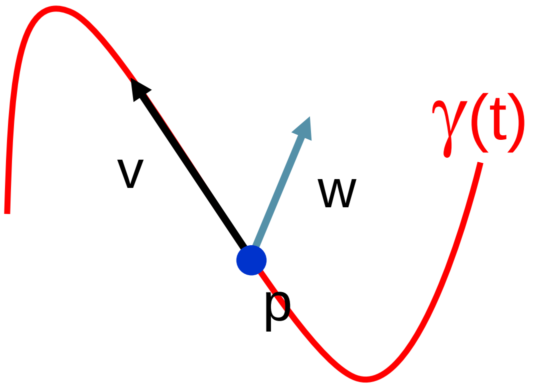
$$< v, w > = \cos(\alpha) \|v\| \|w\|$$

- Length of a curve

$$L(\gamma) = \int \| \dot{\gamma}(t) \| dt$$







Measuring extrinsic distances

Basic tool: the scalar product

Bernhard Riemann
1826-1866

$$<\boldsymbol{v},\boldsymbol{w}>_p = v^t w = v^t G(p) w$$

- Norm of a vector

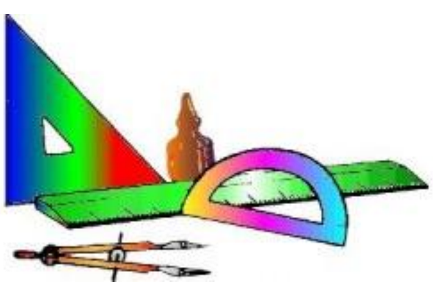
$$\|v\|_p = \sqrt{<v,v>_p}$$

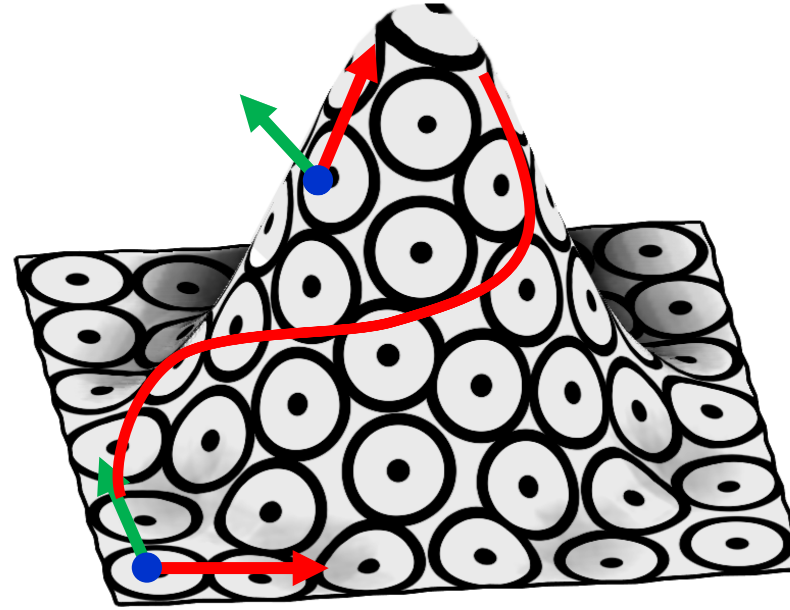
- Angle between vectors

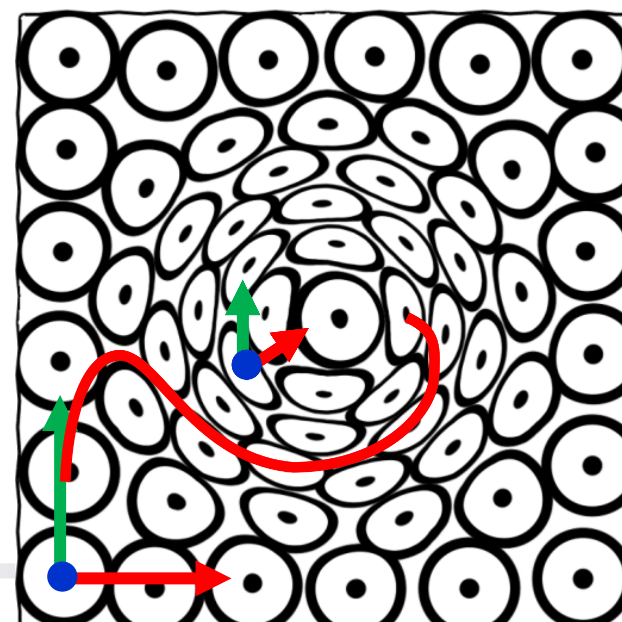
$$<v,w>_p = \cos(\alpha)\,\|v\|_p\,\|w\|_p$$

- Length of a curve

$$L(\gamma) = \int \|\dot{\gamma}(t)\|_{\gamma(t)}\,dt$$





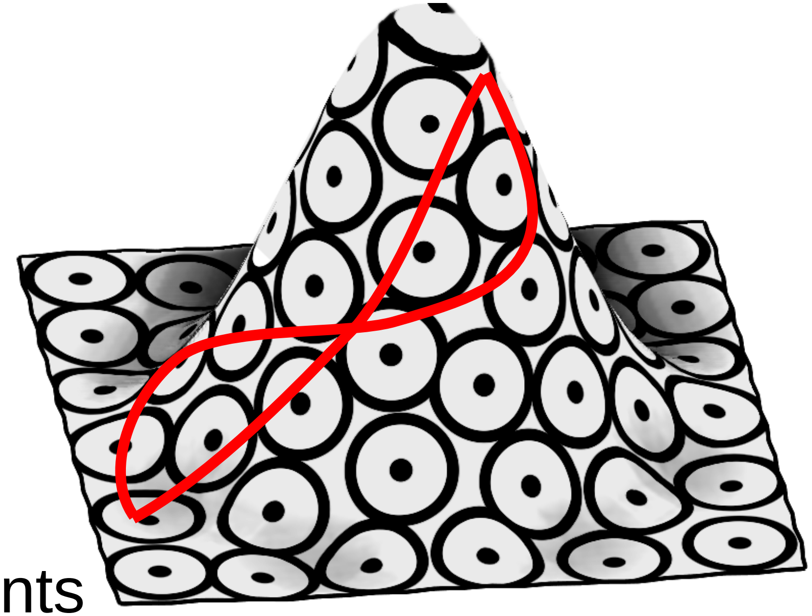


Riemannian manifolds

Basic tool: the scalar product

Bernhard Riemann
1826-1866

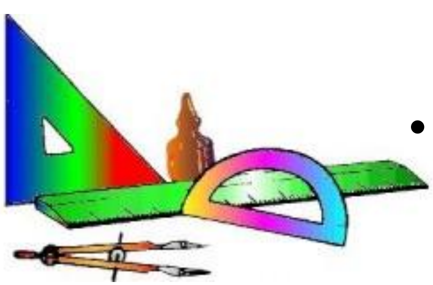
$$< v, w >_p = v^t G(p) w$$

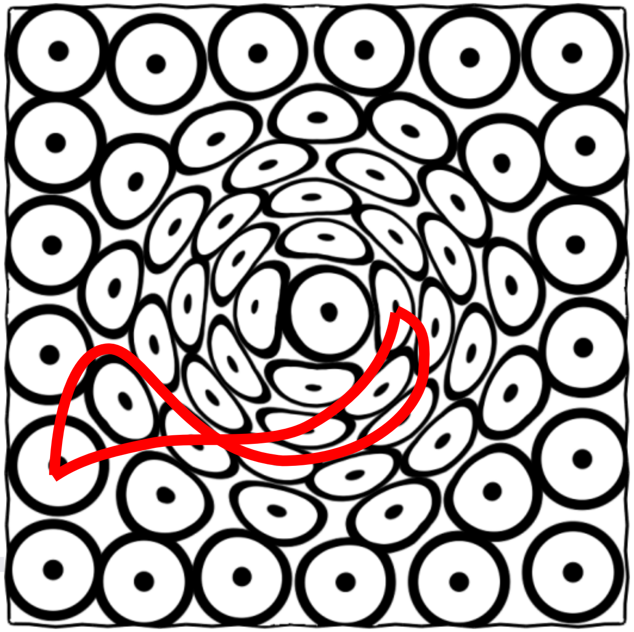


- Geodesic between 2 points
  - Shortest path
  - Calculus of variations (E.L.) : 2^{nd} order differential equation (specifies acceleration)
  - Free parameters: initial speed and starting point

- Length of a curve

$$L(\gamma) = \int \| \dot{\gamma}(t) \|_{\gamma(t)} \, dt$$

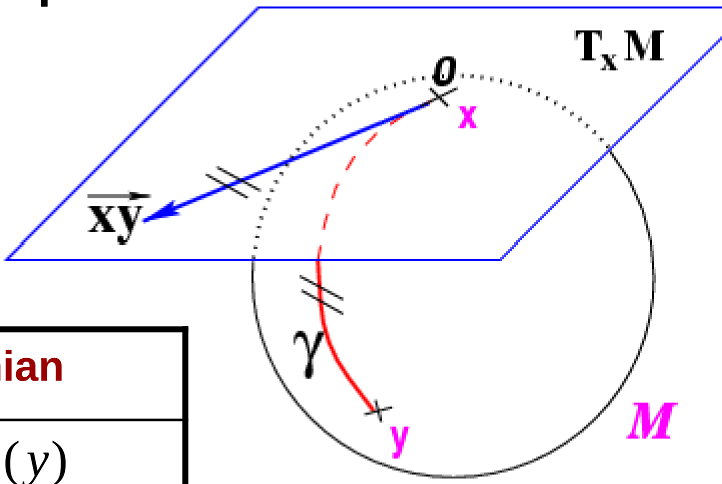




Bases of Algorithms in Riemannian Manifolds

Exponential map (Normal coordinate system):

- **Exp_x** = geodesic shooting parameterized by the initial tangent
- **Log_x** = unfolding the manifold in the tangent space along geodesics
  - Geodesics = straight lines with Euclidean distance
  - Local → global domain: star-shaped, limited by the cut-locus
  - Covers all the manifold if **geodesically complete**



Reformulate algorithms with exp_x and log_x

Vector -> Bi-point (no more equivalence classes)

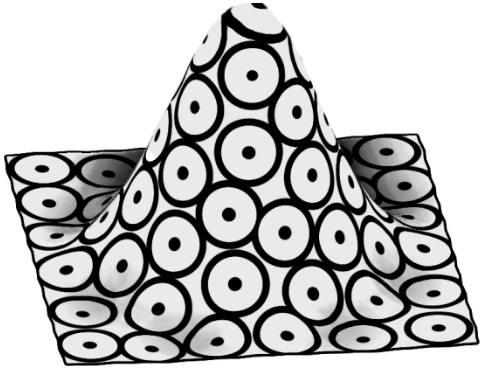
Operation	Euclidean space	Riemannian
Subtraction	$\overrightarrow{xy} = y - x$	$\overrightarrow{xy} = Log_x(y)$
Addition	$y = x + \overrightarrow{xy}$	$y = Exp_x(\overrightarrow{xy})$
Distance	$\text{dist}(x, y) = \lVert y - x \rVert$	$\text{dist}(x, y) = \lVert \overrightarrow{xy} \rVert_x$
Gradient descent	$x_{t+\varepsilon} = x_t - \varepsilon \nabla C(x_t)$	$x_{t+\varepsilon} = Exp_{x_t}(-\varepsilon \nabla C(x_t))$

Random variable in a Riemannian Manifold

Intrinsic pdf of x

- For every set H

$$P(\mathbf{x} \in H) = \int_H p(y)dM(y)$$



- ~~Lebesgue's measure~~

→ Uniform Riemannian Mesure $dM(y) = \sqrt{\det(G(y))}\, dy$

Expectation of an observable in M

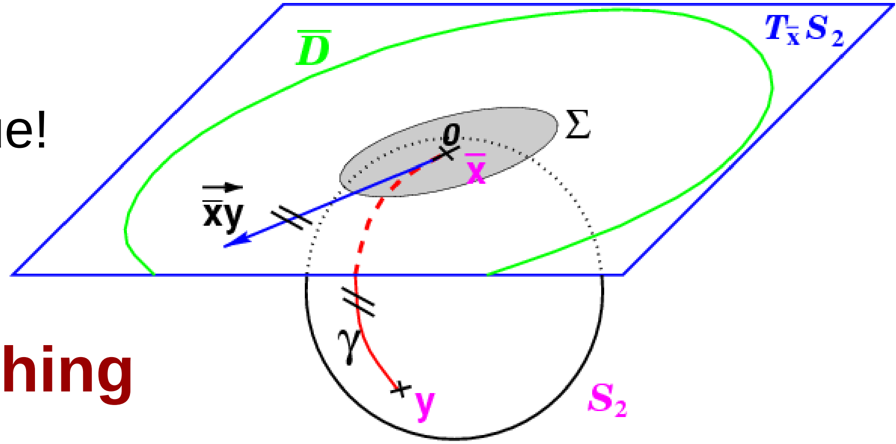
- $\boldsymbol{E}_{\mathbf{x}}[\phi] = \int_M \phi(y)p(y)dM(y)$
- $\phi = dist^2$ (variance) : $\boldsymbol{E}_{\mathbf{x}}[dist(.,y)^2] = \int_M dist(y,z)^2 p(z)dM(z)$
- $\phi = \log(p)$ (information) : $\boldsymbol{E}_{\mathbf{x}}[\log(p)] = \int_M p(y)\log(p(y))dM(y)$
- ~~$\phi = x$ (mean) : $\boldsymbol{E}_{\mathbf{x}}[\mathbf{x}] = \int_M y\, p(y)dM(y)$~~

First Statistical Tools: Moments

Frechet / Karcher mean minimize the variance

$$\mathrm{E}[\mathbf{x}] = \underset{y \in \mathrm{M}}{\operatorname{argmin}} \left(\mathrm{E}\left[\operatorname{dist}(y, \mathbf{x})^2 \right] \right) \quad \Rightarrow \quad \mathrm{E}\left[\overrightarrow{\bar{\mathrm{x}}\mathbf{x}} \right] = \int_{\mathrm{M}} \overrightarrow{\bar{\mathrm{x}}\mathbf{x}}.p_{\mathbf{x}}(z).d\mathrm{M}(z) = 0 \quad [P(C) = 0]$$

- Variational characterization: Exponential barycenters
- Existence and uniqueness (convexity radius) [Karcher / Kendall / Le / Afsari]
- Empirical mean: almost surely unique! [Arnaudon & Miclo 2013]



Gauss-Newton Geodesic marching

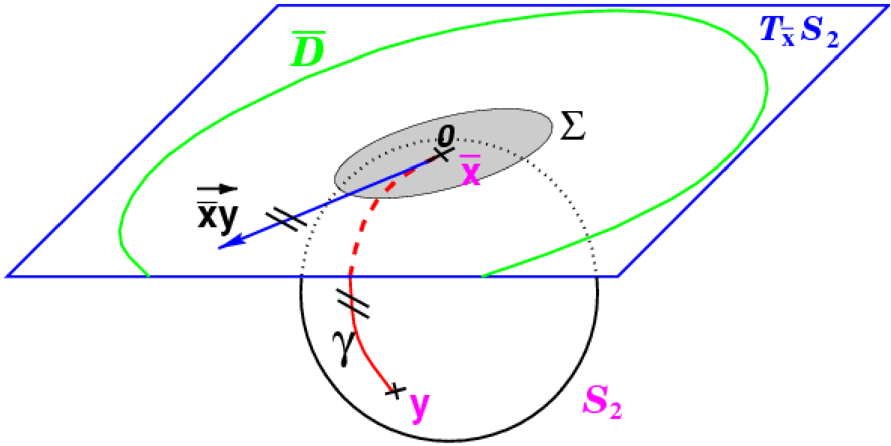
$$\bar{\mathrm{x}}_{t+1} = \exp_{\bar{\mathrm{x}}_t}(v) \quad \text{with} \quad v = \mathrm{E}\left[\overrightarrow{\mathrm{y}\mathbf{x}} \right] = \frac{1}{n} \sum_{i=1}^{n} \mathrm{Log}_{\bar{\mathrm{x}}_t}(\mathrm{x}_i)$$

[Oller & Corcuera 95, Battacharya & Patrangenaru 2002, Pennec, JMIV06, NSIP'99]

First Statistical Tools: Moments

Covariance (PCA) [higher moments]

$$\Sigma_{\mathbf{xx}} = \mathrm{E}\left[\left(\overrightarrow{\bar{\mathrm{x}}\mathbf{x}}\right).\left(\overrightarrow{\bar{\mathrm{x}}\mathbf{x}}\right)^{\mathrm{T}}\right] = \int_{\mathrm{M}} \left(\overrightarrow{\bar{\mathrm{x}}z}\right).\left(\overrightarrow{\bar{\mathrm{x}}z}\right)^{\mathrm{T}} .p_{\mathbf{x}}(z).d\mathrm{M}(z)$$



Principal component analysis

- Tangent-PCA:
  principal modes of the covariance
- Principal Geodesic Analysis (PGA) [Fletcher 2004,Sommer 2014]

[Oller & Corcuera 95, Battacharya & Patrangenaru 2002, Pennec, JMIV06, NSIP'99]

Distributions for parametric tests

Generalization of the Gaussian density:

- Stochastic heat kernel p(x,y,t) [complex time dependency]
- Wrapped Gaussian [Infinite series difficult to compute]
- Maximal entropy knowing the mean and the covariance

$$N(y) = k.\exp\left(\left(\overrightarrow{\bar{x}\mathbf{x}}\right)^{T} . \boldsymbol{\Gamma} . \left(\overrightarrow{\bar{x}\mathbf{x}}\right) / 2 \right)$$

$$\boldsymbol{\Gamma} = \boldsymbol{\Sigma}^{(-1)} - \tfrac{1}{3}\mathrm{Ric} + O(\sigma) + \varepsilon(\sigma / r)$$

$$k = (2\pi)^{-n/2} . \det(\boldsymbol{\Sigma})^{-1/2} . \left(1 + O(\sigma^{3}) + \varepsilon(\sigma / r)\right)$$

Mahalanobis D2 distance / test:

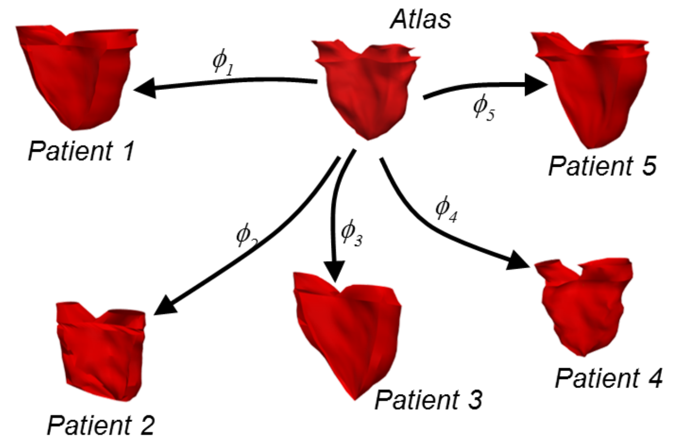
$$\mu_{\mathbf{x}}^{2}(\mathrm{y}) = {\overrightarrow{\bar{x}\mathrm{y}}}^{t} . \Sigma_{\mathbf{xx}}^{(-1)} . \overrightarrow{\bar{x}\mathrm{y}}$$

- Any distribution: $\mathrm{E}\left[\mu_{\mathbf{x}}^{2}(\mathbf{x})\right] = n$
- Gaussian: $\mu_{\mathbf{x}}^{2}(\mathbf{x}) \propto \chi_{n}^{2} + O(\sigma^{3}) + \varepsilon(\sigma / r)$

[Pennec, NSIP'99, JMIV 2006]

When the manifold is a Transformation Group

Anatomical variability through transformations



Lie groups: Smooth manifold G with group structure

- Composition $g \circ h$ and inversion g^{-1} are smooth
- Left and Right translation $L_g(f) = g \circ f \quad R_g(f) = f \circ g$

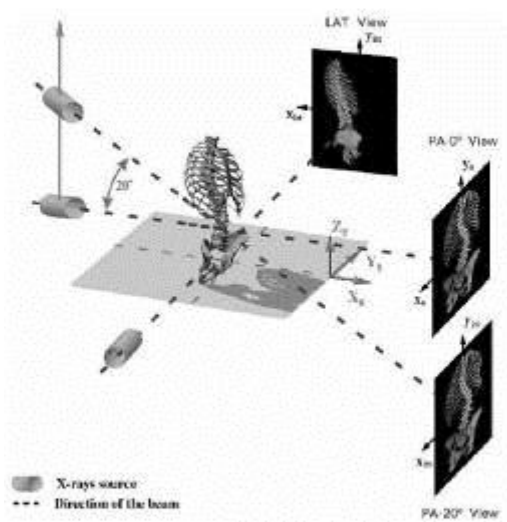
Natural Riemannian metric choices

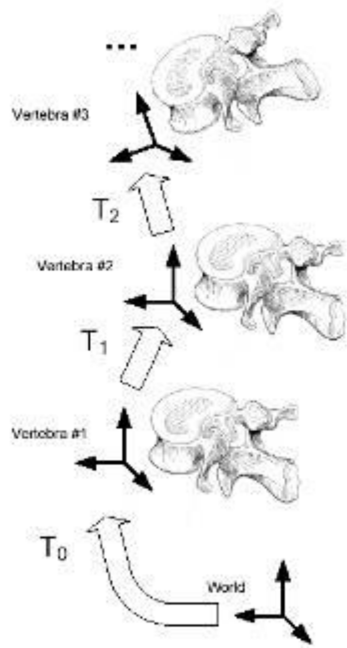
- Chose a metric at Id: $<x,y>_{Id}$
- Propagate at each point g using left (or right) translation
  $<x,y>_g = < DL_{g^{(-1)}}.x \,,\, DL_{g^{(-1)}}.y >_{Id}$
- Simple implementation (homogeneity) using left (resp. right) translation

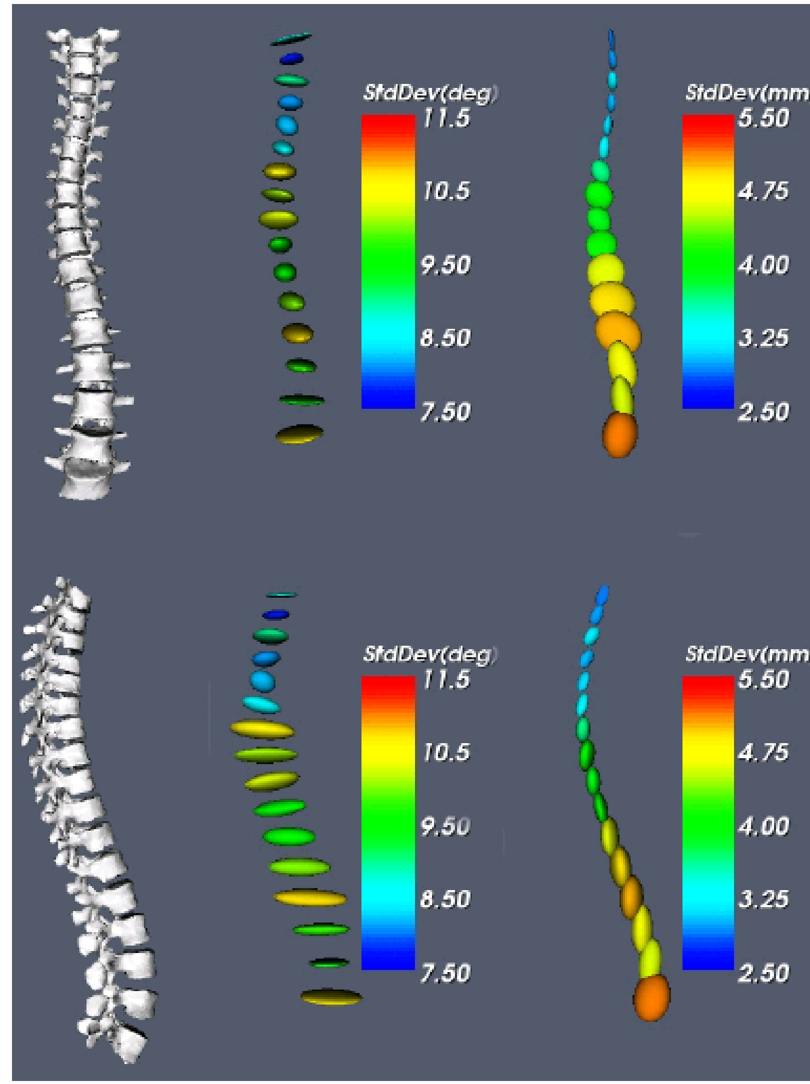
$$\mathrm{Exp}_f(x) = f \circ \mathrm{Exp}_{Id}(DL_{f^{(-1)}}.x) \qquad \overrightarrow{fg} = \mathrm{Log}_f(g) = DL_f.\mathrm{Log}_{Id}(f^{(-1)} \circ g)$$

Statistical Analysis of the Scoliotic Spine

[J. Boisvert et al. ISBI'06, AMDO'06 and IEEE TMI 27(4), 2008]







Database

- 307 Scoliotic patients from the Montreal's Sainte-Justine Hospital.
- 3D Geometry from multi-planar X-rays

Mean

- Main translation variability is axial (growth?)
- Main rot. var. around anterior-posterior axis

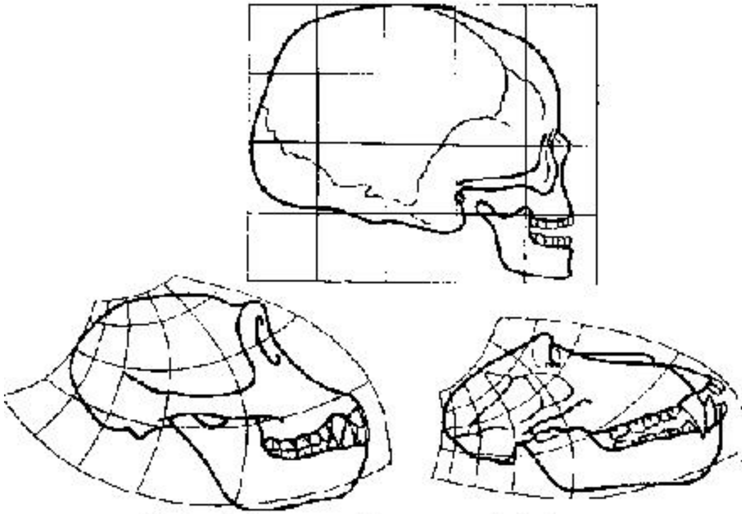
Statistical Analysis of the Scoliotic Spine

[J. Boisvert et al. ISBI'06, AMDO'06 and IEEE TMI 27(4), 2008]
AMDO'06 best paper award, Best French-Quebec joint PhD 2009

PCA of the Covariance:
4 first variation modes have clinical meaning

- Mode 1: King's class I or III
- Mode 2: King's class I, II, III
- Mode 3: King's class IV + V
- Mode 4: King's class V (+II)

Morphometry through Deformations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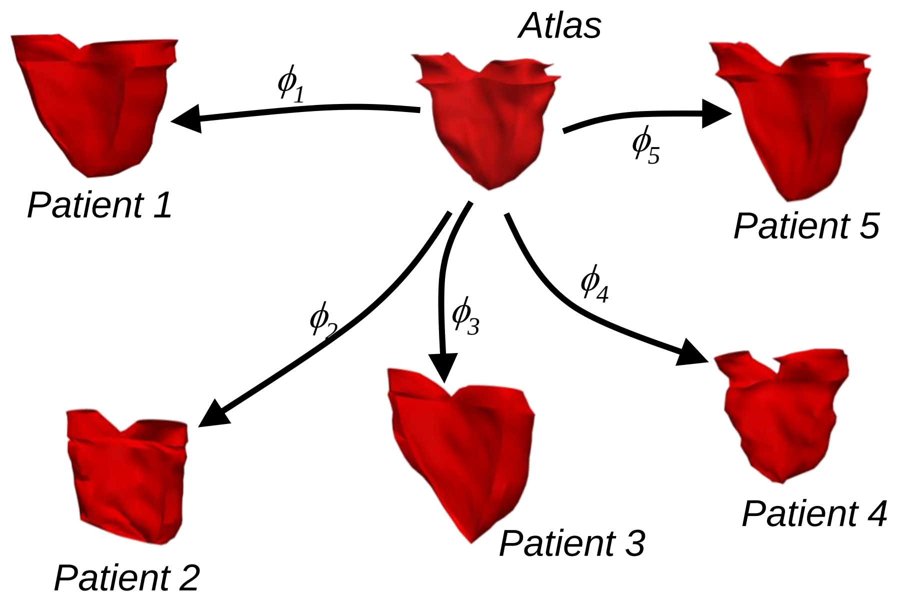


Measure of deformation [D'Arcy Thompson 1917, Grenander & Miller]

- Observation = "random" deformation of a reference template
- Deterministic template = anatomical invariants [Atlas ~ mean]
- Random deformations = geometrical variability [Covariance matrix]

Riemannian Metrics on diffeomorphisms

Space of deformations

- Transformation y=ϕ(x)
- Curves in transformation spaces: ϕ(x,t)
- Tangent vector = speed vector field

$$v_t(x) = \frac{d\phi(x,t)}{dt}$$

Right invariant metric

$$\left\| v_t \right\|_{\phi_t} = \left\| v_t \circ \phi_t^{-1} \right\|_{Id}$$

- Lagrangian formalism
- Sobolev Norm H_k or H_∞ (RKHS) in LDDMM → diffeomorphisms
  [Miller, Trouve, Younes, Holm, Dupuis, Beg… 1998 – 2009]
- Geometric Mechanics [Arnold, Smale, Souriau, Marsden, Ratiu, Holmes, Michor…]

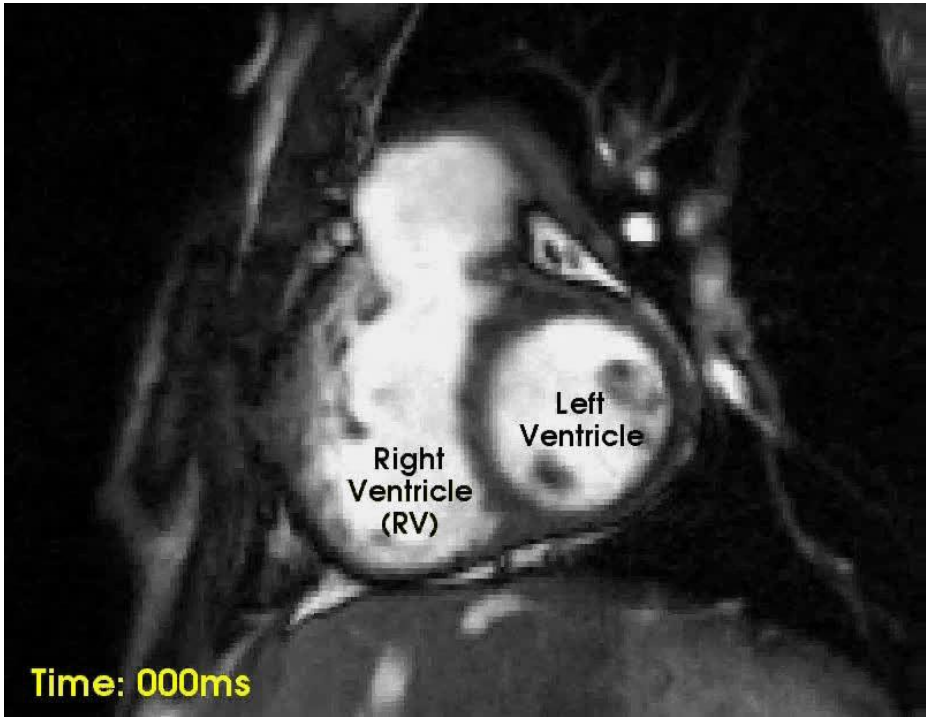
Geodesics determined by optimization of a time-varying vector field

- Distance

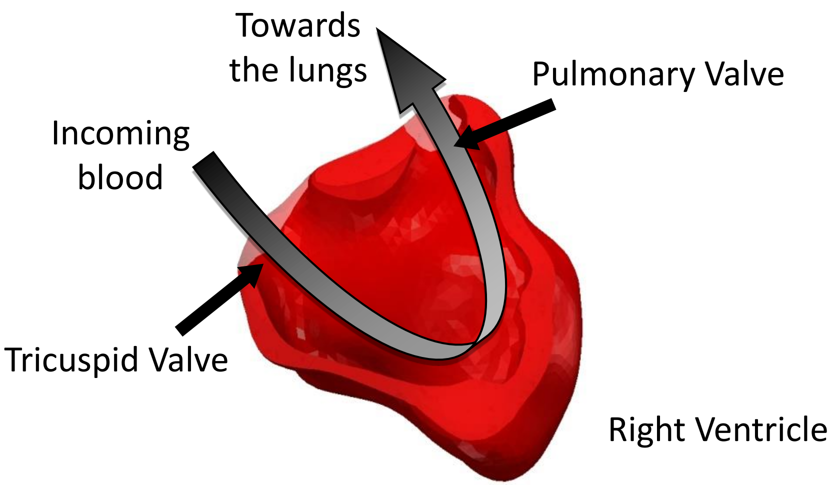
$$d^2(\phi_0, \phi_1) = \arg\min_{v_t}\left(\int_0^1 \left\| v_t \right\|_{\phi_t}^2 dt\right)$$

- Geodesics characterized by initial velocity / momentum
- Optimization by shooting/adjoint or path-straightening methods

Repaired Tetralogy of Fallot



- *Severe Congenital Heart Disease*
- *Occurs 1 of 2500 (Hoffman, JACC 02)*
- *Surgical repair in infancy*
- *After repair: chronic pulmonary valve regurgitations and extremely dilated right ventricle (RV).*



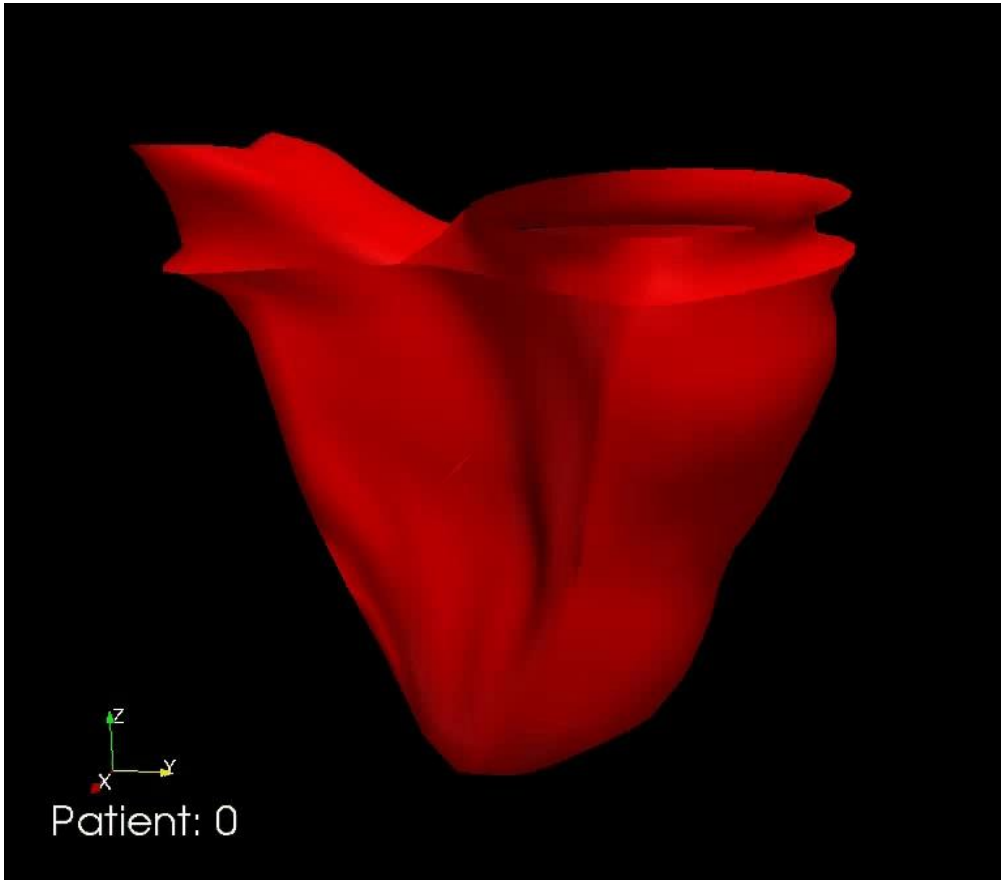
Best time for valve replacement: understand / quantify the remodeling

http://www-sop.inria.fr/asclepios/projects/Health-e-Child/ShapeAnalysis/index.php

Repaired Tetralogy of Fallot

Remodeling of the right ventricle of the heart in tetralogy of Fallot

- Mean shape
- Shape variability
- Correlation with clinical variables
- Predicting remodeling effect

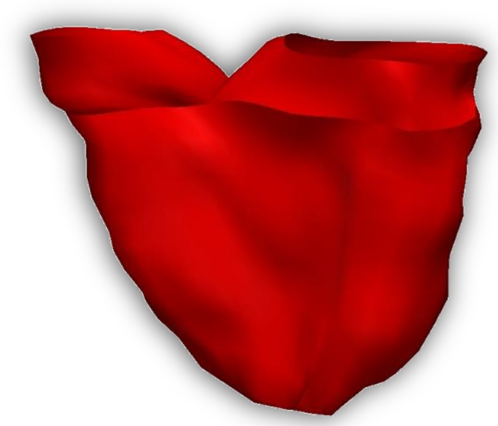


Shape of RV in 18 patients

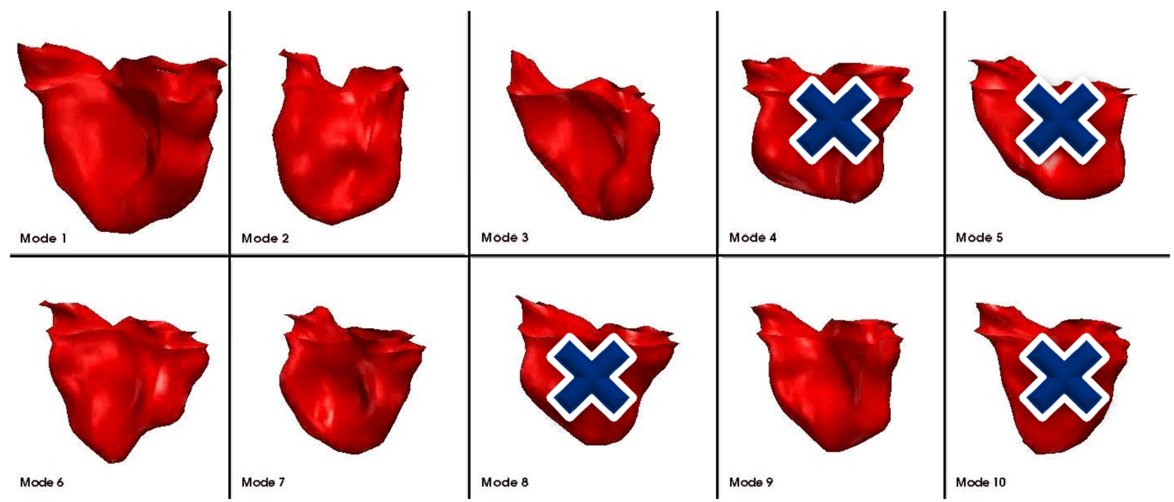
Atlas and Deformations Joint Estimation

Method: PLS (better than PCA + CCA) to

- Find modes that are significantly correlated to clinical variables (body surface area, tricuspid and pulmonary valve regurgitations).
- Create a generative model by regressing shape vs BSA



Average RV anatomy of 18 ToF patients



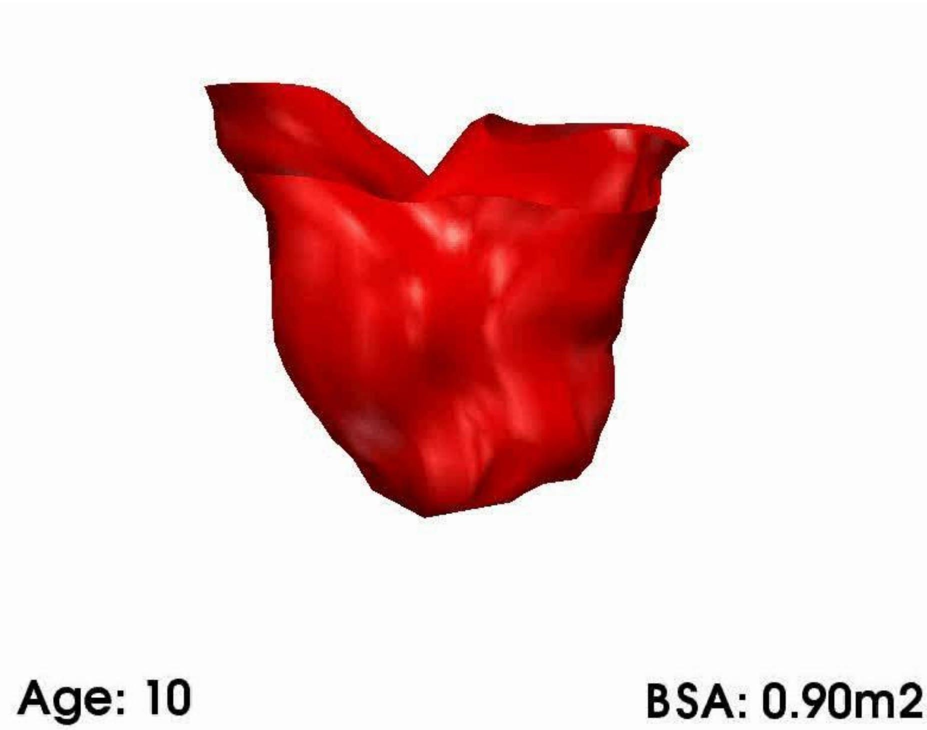
10 Deformation Modes = 90% of spectral energy

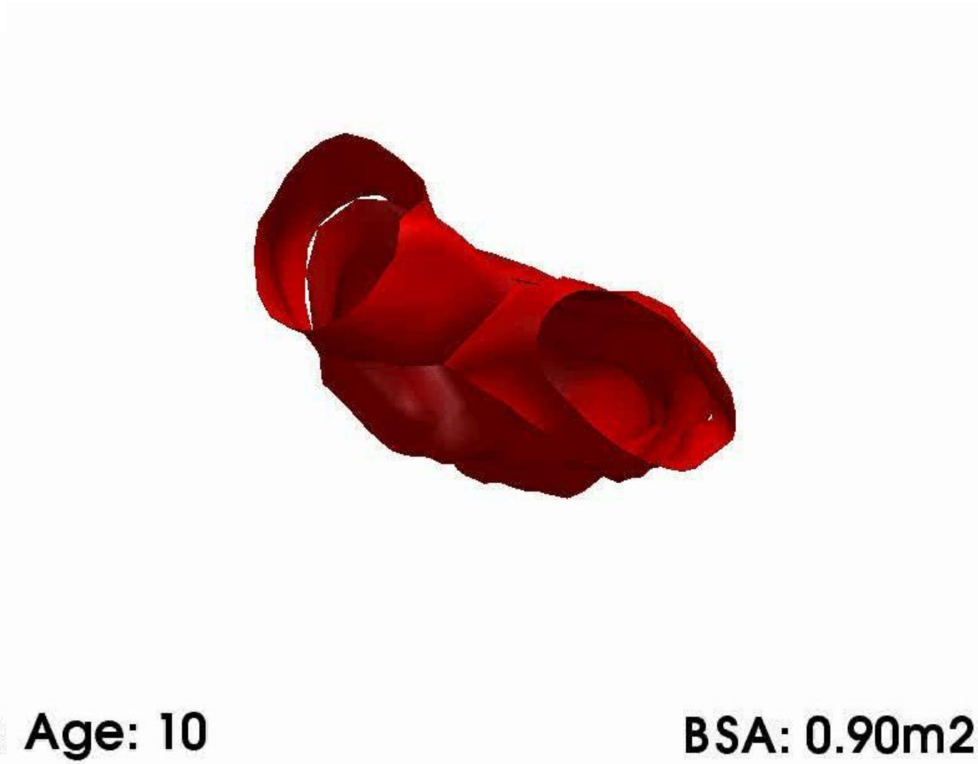
6 modes significantly correlated to BSA

[Mansi et al, MICCAI 2009, TMI 2011]

Statistical Remodeling of RV in Tetralogy of Fallot

[Mansi et al, MICCAI 2009, TMI 2011]





Predicted remodeling effect … has a clinical interpretation

Volume increases → Valve annuli deform → Pulmonary stenosis reduces → RV pressure decreases → Septum pushed inwards → RV free-wall outwards

Limits of the Riemannian Framework

No bi-invariant metric in general

- **Incompatibility of the Fréchet mean with the group structure**
  - Left of right metric: different Fréchet means
  - The inverse of the mean is not the mean of the inverse
- Examples with simple 2D rigid transformations

- **Can we design a mean compatible with the group operations?**
- **Is there a more convenient structure for statistics on Lie groups?**

Outline

Riemannian frameworks on Lie groups

- Manifolds
- Statistics
- Applications to spine shape & heart remodeling

Lie groups as affine connection spaces

- The bi-invariant affine Cartan connection structure
- Extending statistics without a metric

The SVF framework for diffeomorphisms

- Diffeomorphisms with SVFs
- Longitudinal modeling of brain atrophy in AD

Basics of Lie groups

Flow of a left invariant vector field $\tilde{X} = DL.x$ starting from e

- $\gamma_x(t)$ exists for all time
- One parameter subgroup: $\gamma_x(s+t) = \gamma_x(s).\,\gamma_x(t)$

Lie group exponential

- Definition: $x \in \mathfrak{g} \rightarrow Exp(x) = \gamma_x(1) \in G$
- Diffeomorphism from a neighborhood of 0 in $\mathfrak{g}$ to a neighborhood of e in G (not true in general for inf. dim)
- Baker-Campbell Hausdorff (BCH) formula

$$BCH(x,y) = Log\big(Exp(x).Exp(y)\big) = x + y + \frac{1}{2}[x,y] + \ \ldots$$

3 curves at each point parameterized by the same tangent vector

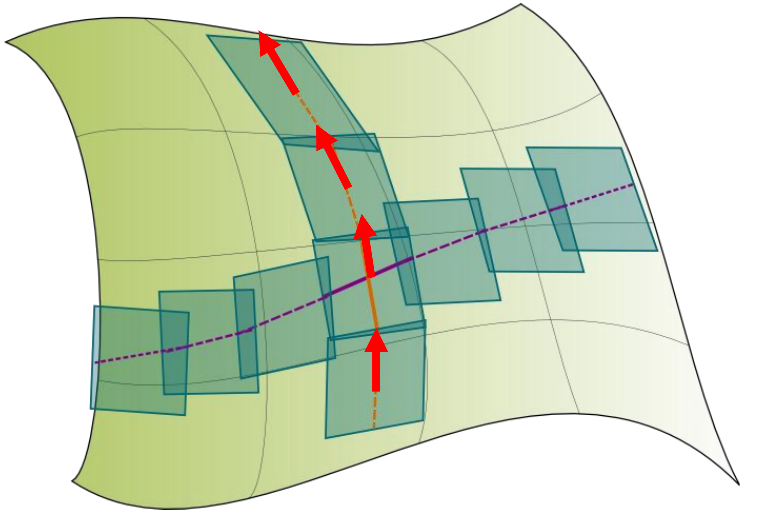
- Left / Right-invariant geodesics, one-parameter subgroups

Question: Can one-parameter subgroups be geodesics?

Affine connection spaces

Affine Connection (infinitesimal parallel transport)

- Acceleration = derivative of the tangent vector along a curve
- Projection of a tangent space on a neighboring tangent space



Adapted from Lê Nguyên Hoang, science4all.org

Geodesics = straight lines

- Null acceleration: $\nabla_{\dot{\gamma}}\dot{\gamma} = 0$
- 2nd order differential equation: Normal coordinate system
- Local exp and log maps

Cartan-Schouten Connection on Lie Groups

A unique connection

- Symmetric (no torsion) and bi-invariant
- For which geodesics through Id are one-parameter subgroups (group exponential)
  - Matrices : M(t) = A.exp(t.V)
  - Diffeos : **translations of Stationary Velocity Fields (SVFs)**

Levi-Civita connection of a bi-invariant metric (if it exists)

- Continues to exists in the absence of such a metric (e.g. for rigid or affine transformations)

Two flat connections (left and right)

- **Absolute parallelism**: no curvature but torsion (Cartan / Einstein)

Statistics on an affine connection space

~~Fréchet mean~~: exponential barycenters

- $\sum_i Log_x(y_i) = 0$ [Emery, Mokobodzki 91, Corcuera, Kendall 99]
- Existence & **local uniqueness** if local convexity [Arnaudon & Li, 2005]

For Cartan-Schouten connections [Pennec & Arsigny, 2012]

- Locus of points *x* such that $\sum Log(x^{-1}.y_i) = 0$
- Algorithm: fixed point iteration (**local convergence**)

$$x_{t+1} = x_t \circ Exp\left(\frac{1}{n}\sum Log(x_t^{-1}.y_i)\right)$$

- **Mean stable by left / right composition and inversion**
  - If m is a mean of $\{g_i\}$ and h is any group element, then $h \circ m$ is a mean of $\{h \circ g_i\}$, $m \circ h$ is a mean of the points $\{g_i \circ h\}$ and $m^{(-1)}$ is a mean of $\{g_i^{(-1)}\}$

Special matrix groups

Heisenberg Group (resp. Scaled Upper Unitriangular Matrix Group)

- No bi-invariant metric
- Group geodesics defined globally, all points are reachable
- Existence and uniqueness of bi-invariant mean (closed form resp. solvable)

Rigid-body transformations

- Logarithm well defined iff log of rotation part is well defined, i.e. if the 2D rotation have angles $|\theta_i| < \pi$
- Existence and uniqueness with same criterion as for rotation parts (same as Riemannian)

Invertible linear transformations

- Logarithm unique if no complex eigenvalue on the negative real line
- Generalization of geometric mean (as in LE case but different)

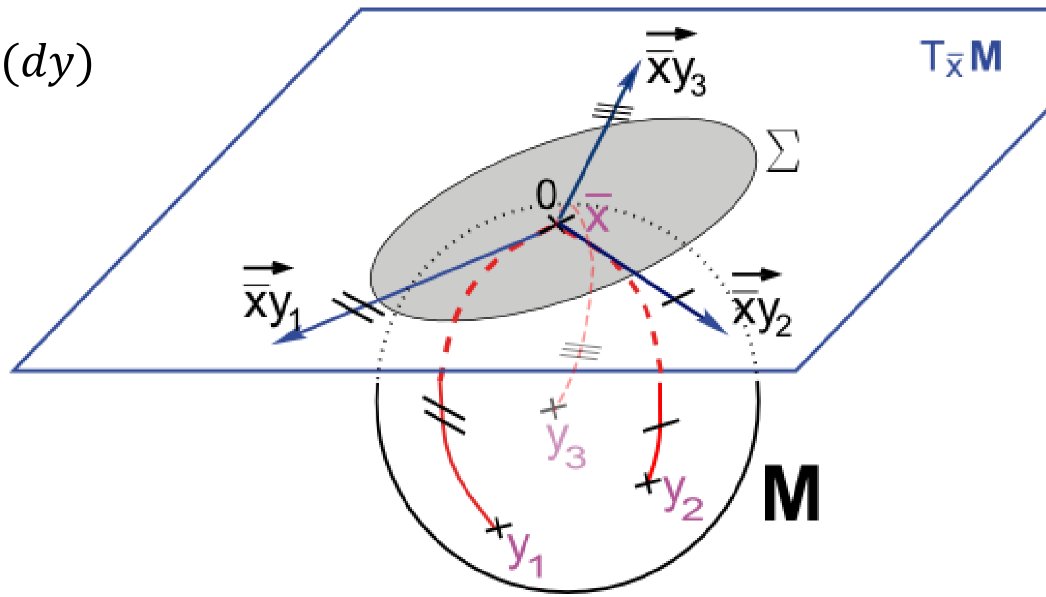
Generalization of the Statistical Framework

Co[redacted]ariance matrix & higher order moments

- Defined as tensors in tangent space

$$\Sigma = \int Log_x(y) \otimes Log_x(y)\, \mu(dy)$$

- Matrix expression changes [redacted]ccording to the basis



Oth[redacted]r statistical tools

- Mahalanobis distance well defined and bi-invariant
- ~~Tangent Principal Component Analysis (t-PCA)~~
- Principal Geodesic Analysis (PGA), provided a data likelihood
- Independent Component Analysis (ICA)

Cartan Connections vs Riemannian

What is similar

- Standard differentiable geometric structure [curved space without torsion]
- Normal coordinate system with Exp_x et Log_x [finite dimension]

Limitations of the affine framework

- No metric (but no choice of metric to justify)
- The exponential does always not cover the full group
  - Pathological examples close to identity in finite dimension
  - In practice, similar limitations for the discrete Riemannian framework

What we gain

- A globally invariant structure invariant by composition & inversion
- Simple geodesics, efficient computations (stationarity, group exponential)
- The simplest linearization of transformations for statistics?

Outline

Riemannian frameworks on Lie groups

- Manifolds
- Statistics
- Applications to spine shape & heart remodeling

Lie groups as affine connection spaces

- The bi-invariant affine Cartan connection structure
- Extending statistics without a metric

The SVF framework for diffeomorphisms

- Diffeomorphisms with SVFs
- Longitudinal modeling of brain atrophy in AD

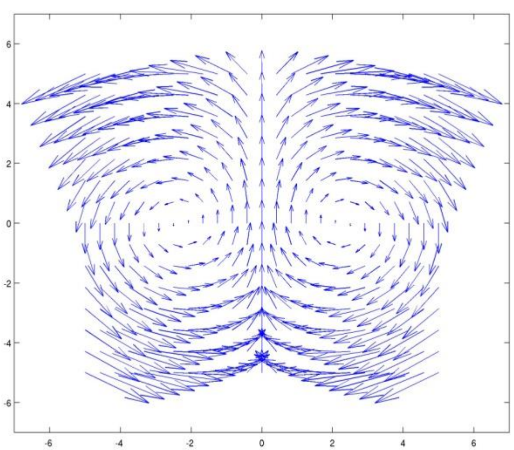
The SVF framework for Diffeomorphisms

Framework of [Arsigny et al., MICCAI 06]

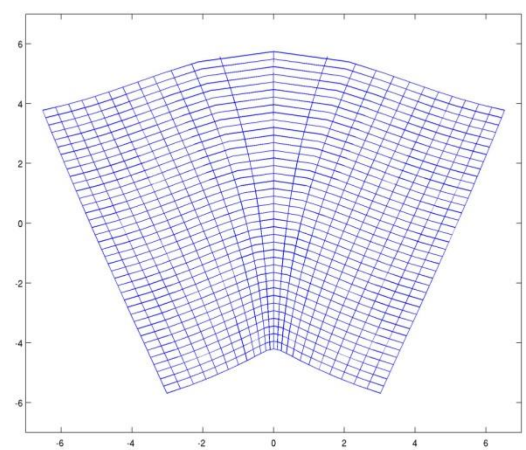
- Use one-parameter subgroups

Exponential of a smooth vector field is a diffeomorphism

- ***u*** is a smooth **stationary velocity field**
- Exponential: solution at time 1 of ODE $\partial x(t) / \partial t = \boldsymbol{u}(x(t))$



Stationary velocity field



Diffeomorphism

The SVF framework for Diffeomorphisms

Efficient numerical methods

- Take advantage of algebraic properties of exp and log.
  - exp(t.V) is a one-parameter subgroup.

$\longrightarrow$ Direct generalization of numerical matrix algorithms.

Efficient parametric diffeomorphisms

- Computing the deformation: Scaling and squaring
  recursive use of $\exp(\boldsymbol{v}) = \exp(\boldsymbol{v}/2) \circ \exp(\boldsymbol{v}/2)$
  [Arsigny MICCAI 2006]

- Updating the deformation parameters:
  BCH formula **[Bossa MICCAI 2007]**

  $$\exp(\boldsymbol{v}) \circ \exp(\varepsilon \boldsymbol{u}) = \exp(\ \boldsymbol{v} + \varepsilon \boldsymbol{u} + [\boldsymbol{v}, \varepsilon \boldsymbol{u}]/2 + [\boldsymbol{v},[\boldsymbol{v}, \varepsilon \boldsymbol{u}]]/12 + \ldots\ )$$

  - Lie bracket $[\boldsymbol{v},\boldsymbol{u}](p) = \mathrm{Jac}(\boldsymbol{v})(p).\boldsymbol{u}(p) - \mathrm{Jac}(\boldsymbol{u})(p).\boldsymbol{v}(p)$

Symmetric log-demons [Vercauteren MICCAI 08]

Idea: [Arsigny MICCAI 2006, Bossa MICCAI 2007, Ashburner Neuroimage 2007]

- Parameterize the deformation by SVFs
- Time varying (LDDMM) replaced by stationary vector fields
- Efficient scaling and squaring methods to integrate autonomous ODEs

Log-demons with SVFs

$$\mathcal{E}(\mathbf{v}, \mathbf{v}_c) = \underbrace{\frac{1}{\sigma_i^2} \|F - M \circ \exp(\mathbf{v}_c)\|_{L_2}^2}_{\text{Similarity}} + \underbrace{\frac{1}{\sigma_x^2} \| \log(\exp(-\mathbf{v}) \circ \exp(\mathbf{v}_c))\|_{L_2}^2}_{\text{Coupling}} + \underbrace{\mathcal{R}(\mathbf{v})}_{\text{Regularisation}}$$

Similarity: *Measures how much the two images differ*

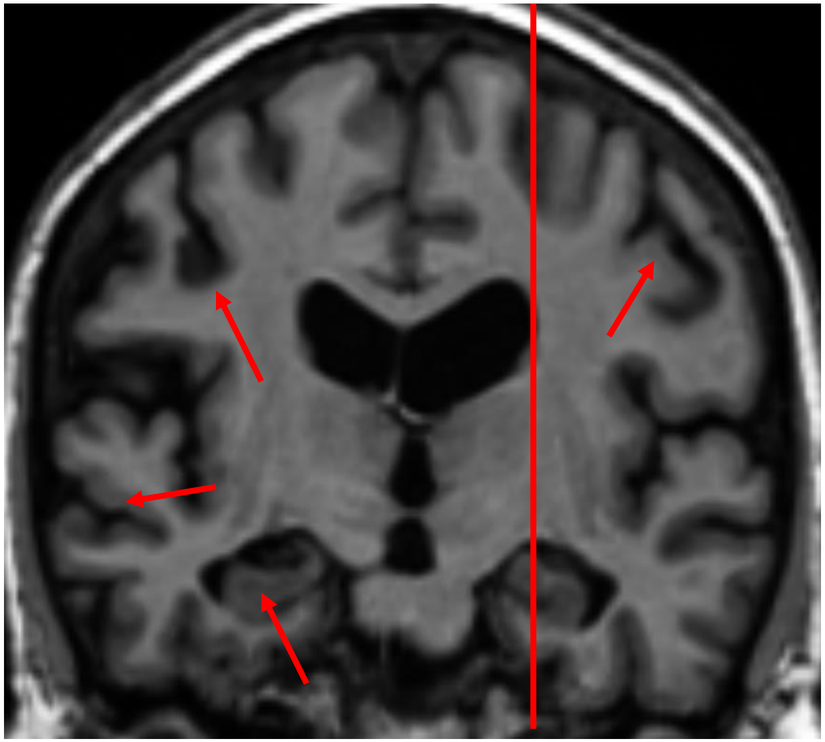
Coupling: *Couples the correspondences with the smooth deformation*

Regularisation: *Ensures deformation smoothness*

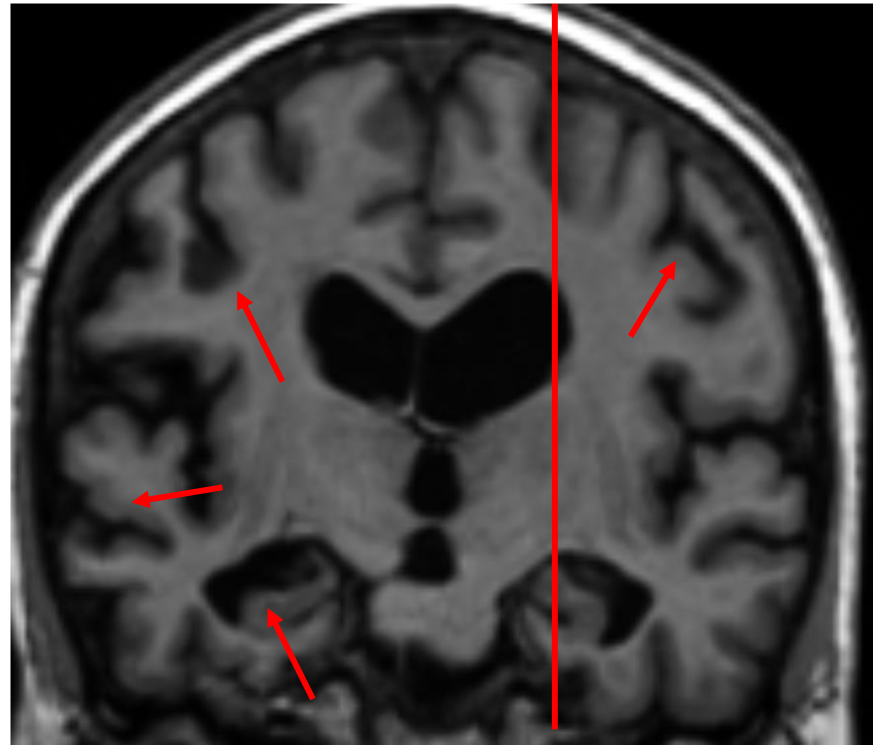
- Efficient optimization with BCH formula
- Inverse consistent with symmetric forces
- Open-source ITK implementation
  - Very fast
  - http://hdl.handle.net/10380/3060

[T Vercauteren, et al.. *Symmetric Log-Domain Diffeomorphic Registration: A Demons-based Approach*, MICCAI 2008]

Longitudinal structural damage in Alzheimer's Disease



baseline

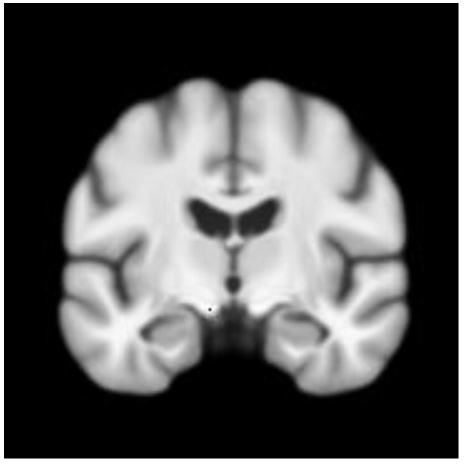


2 years follow-up

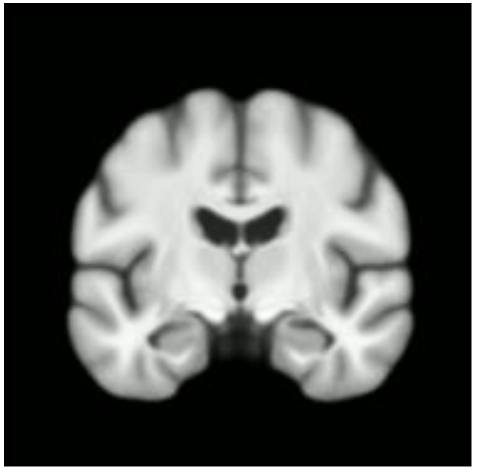
Widespread cortical thinning

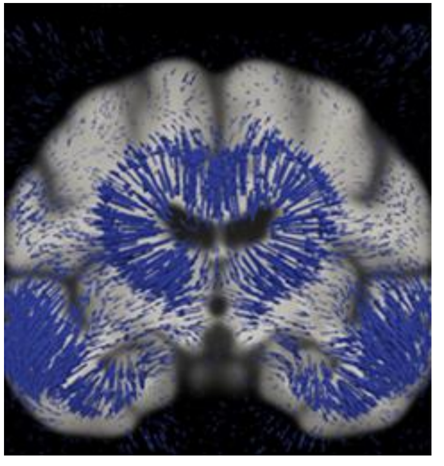
Measuring Temporal Evolution with deformations

Fast registration with deformation parameterized by SVF



$$\varphi_t(x) = exp(t.v(x))$$



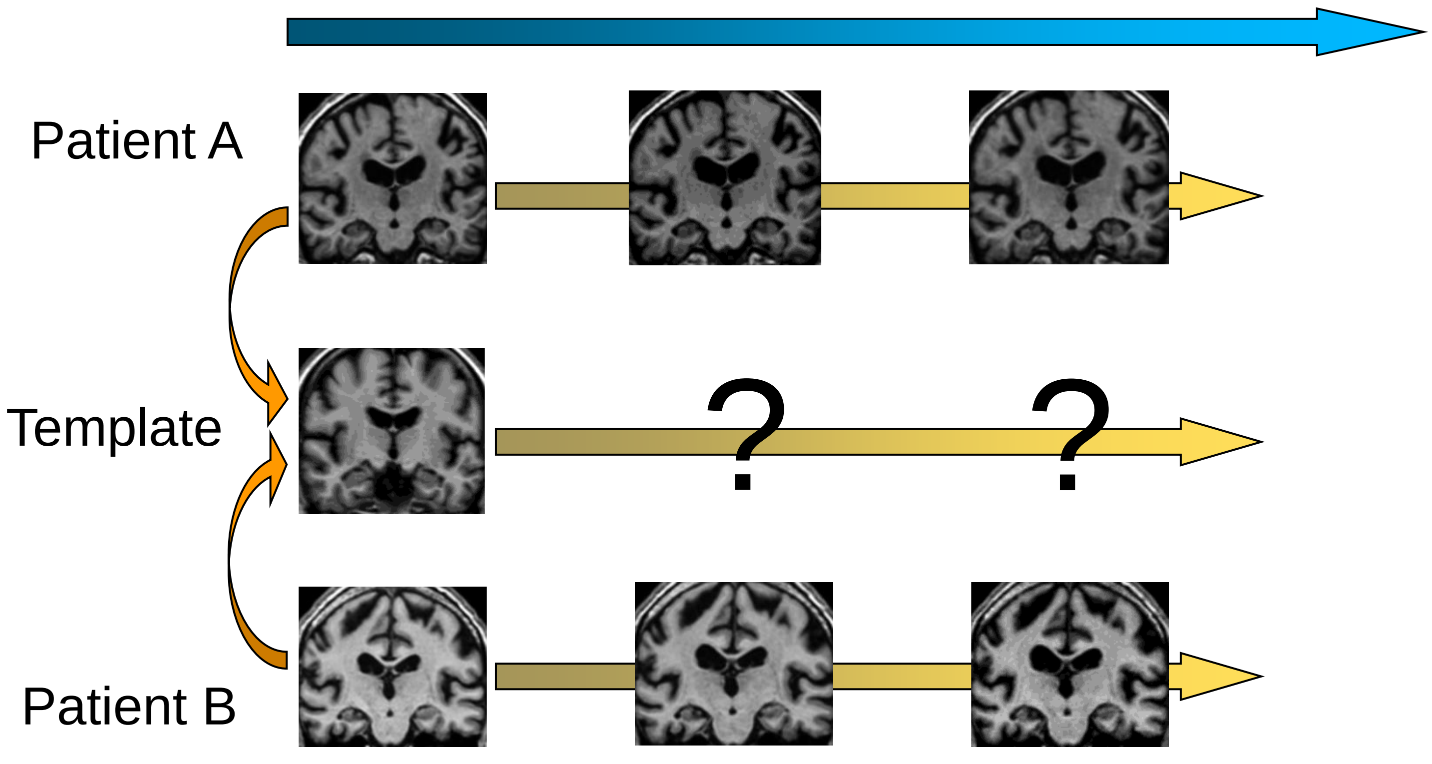


https://team.inria.fr/asclepios/software/lcclogdemons/

[Lorenzi, Ayache, Frisoni, Pennec, Neuroimage 81, 1 (2013) 470-483]

Longitudinal deformation analysis in AD

- From patient specific evolution to population trend (parallel transport of deformation trajectories)
- Inter-subject and longitudinal deformations are of different nature and might require different deformation spaces/metrics



PhD Marco Lorenzi - Collaboration With G. Frisoni (IRCCS FateBenefratelli, Brescia)

Parallel transport of deformations

Encode longitudinal deformation by its initial tangent (co-) vector

- Momentum (LDDMM) / SVF

Parallel transport

- (small) longitudinal deformation vector
- along the large inter-subject normalization deformation

Existing methods

- Vector reorientation with Jacobian of inter-subject deformation
- Conjugate action on deformations (Rao et al. 2006)
- Resampling of scalar maps (Bossa et al, 2010)
- LDDMM setting: parallel transport along geodesics via Jacobi fields [Younes et al. 2008]

Intra and inter-subject deformations/metrics are of different nature

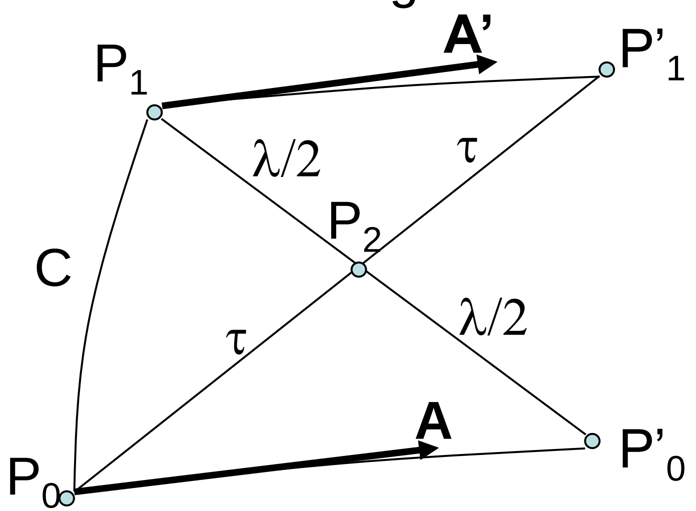
Parallel transport along arbitrary curves

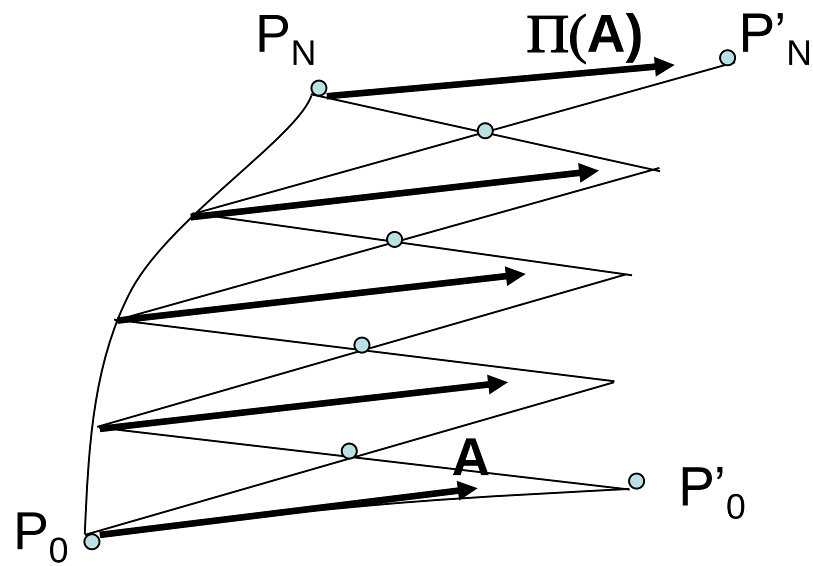
Infinitesimal parallel transport = connection

$\nabla_{\gamma'}(X) : TM \rightarrow TM$

A numerical scheme to integrate for symmetric connections: Schild's Ladder [Elhers et al, 1972]

- Build geodesic parallelogrammoid
- Iterate along the curve

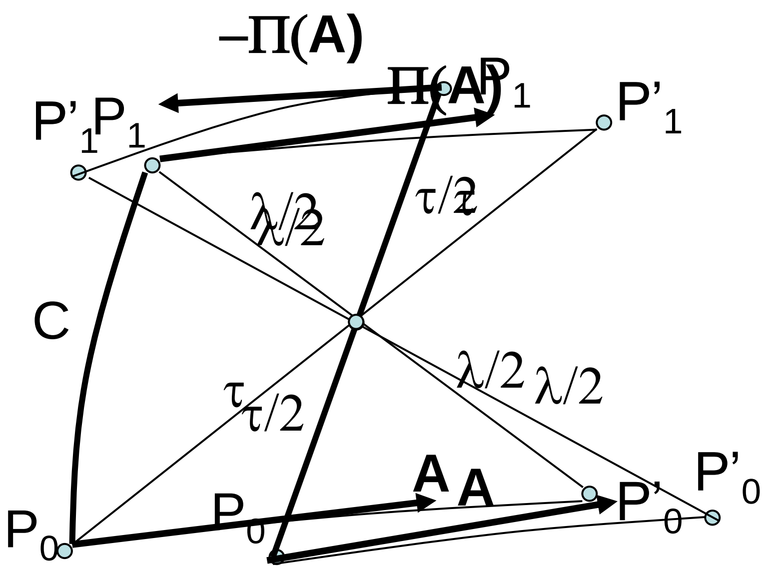


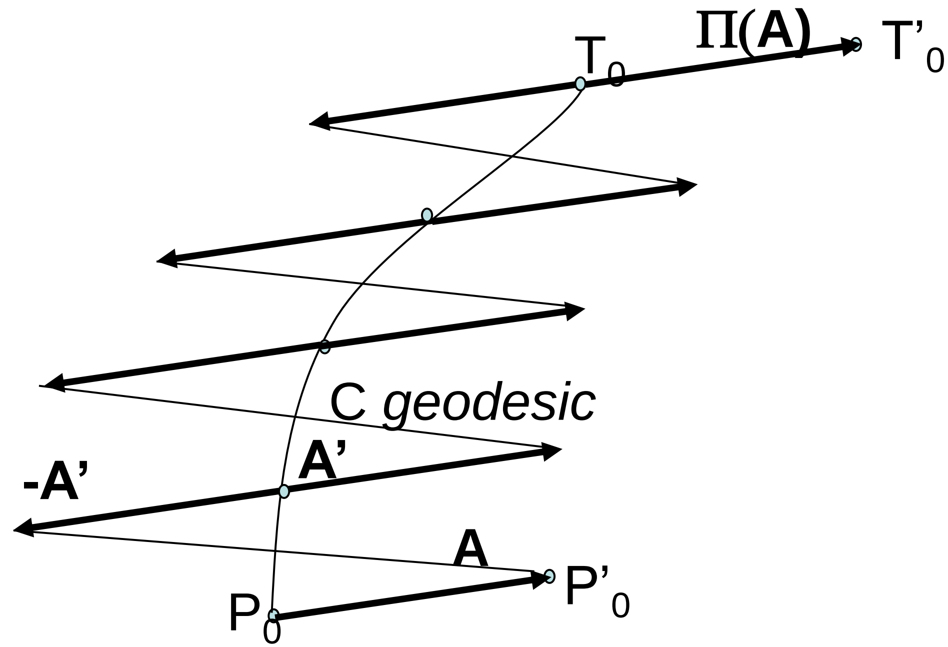


[Lorenzi, Ayache, Pennec: Schild's Ladder for the parallel transport of deformations in time series of images, IPMI 2011]

Parallel transport along geodesics

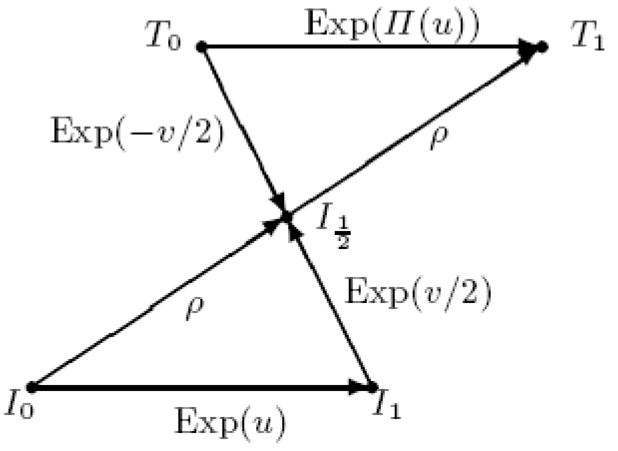
Along geodesics: Pole Ladder [Lorenzi et al, JMIV 2013]





[Lorenzi, Pennec: Efficient Parallel Transport of Deformations in Time Series of Images: from Schild's to pole Ladder, JMIV 2013, to appear]

Efficient Pole and Schild's Ladder with SVFs



$$\mathrm{Exp}(\Pi(u)) = \mathrm{Exp}(v/2) \circ \mathrm{Exp}(u) \circ \mathrm{Exp}(-v/2)$$

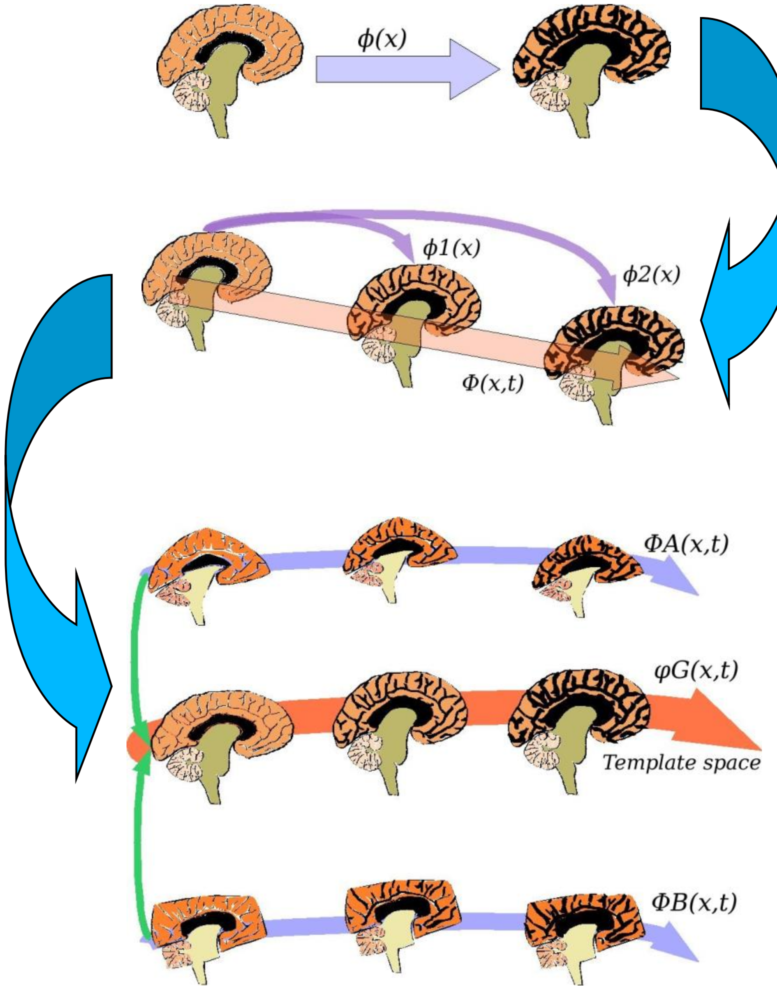
Numerical scheme

- Direct computation $\Pi_{conj}(u) = \mathrm{D}\left(\mathrm{Exp}(v)\right)|_{\mathrm{Exp}(-v)} \cdot u \circ \mathrm{Exp}(-v)$

- Using the BCH: $\Pi_{BCH}(u) = u + [v,u] + \frac{1}{2}[v[v,u]]$

[Lorenzi, Ayache, Pennec: Schild's Ladder for the parallel transport of deformations in time series of images, IPMI 2011]
Runner-up for the IPMI Erbsmann 2011 prize

Analysis of longitudinal datasets Multilevel framework



Single-subject, two time points

Log-Demons (LCC criteria)

Single-subject, multiple time points

4D registration of time series within the Log-Demons registration.

Multiple subjects, multiple time points

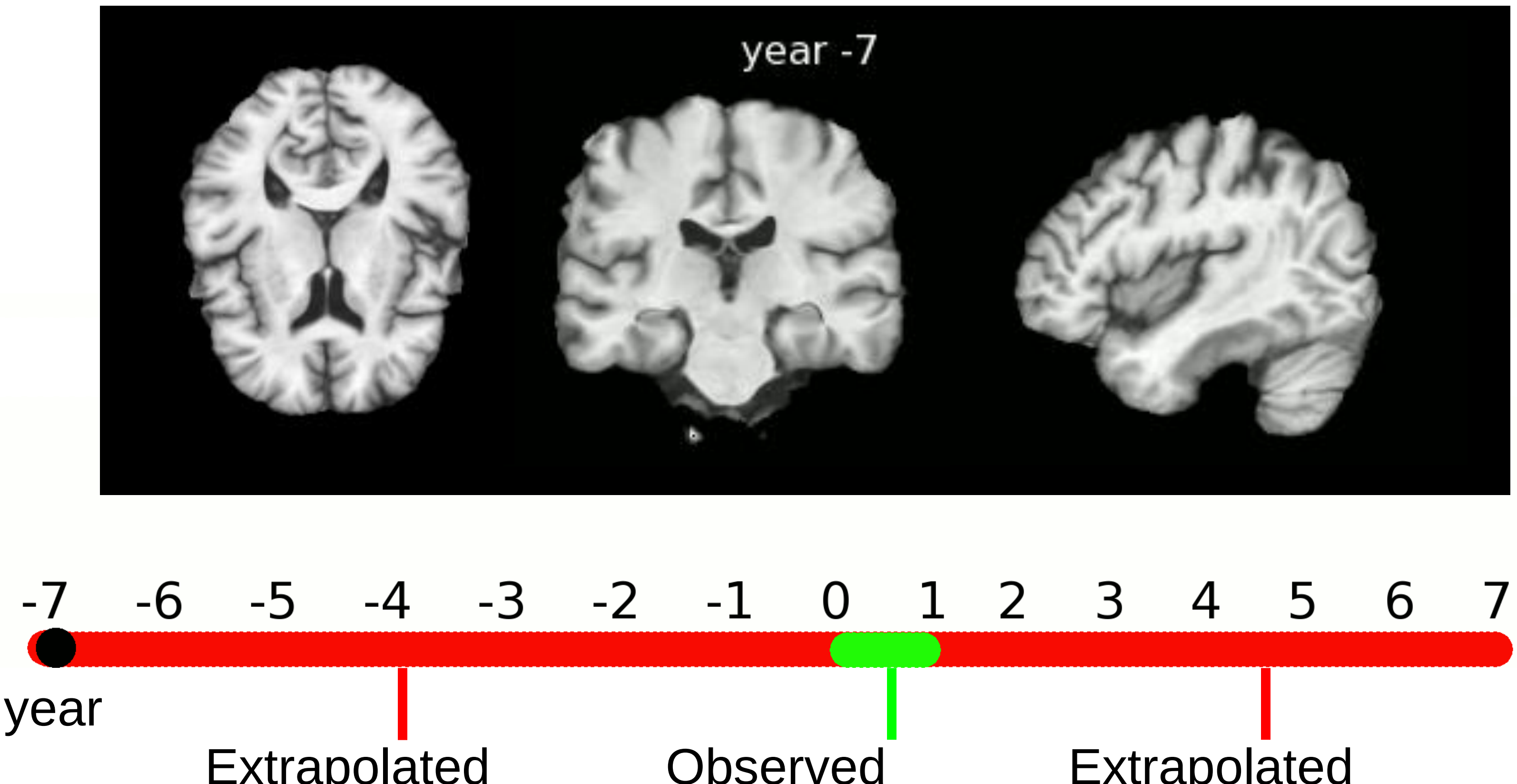
Pole or Schild's Ladder

[Lorenzi et al, in Proc. of MICCAI 2011]

Longitudinal model for AD

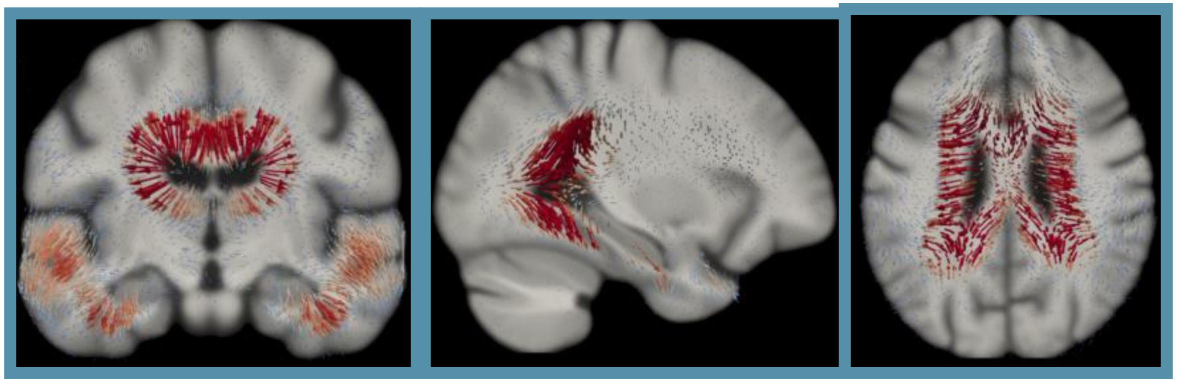
Estimated from 1 year changes – Extrapolation to 15 years

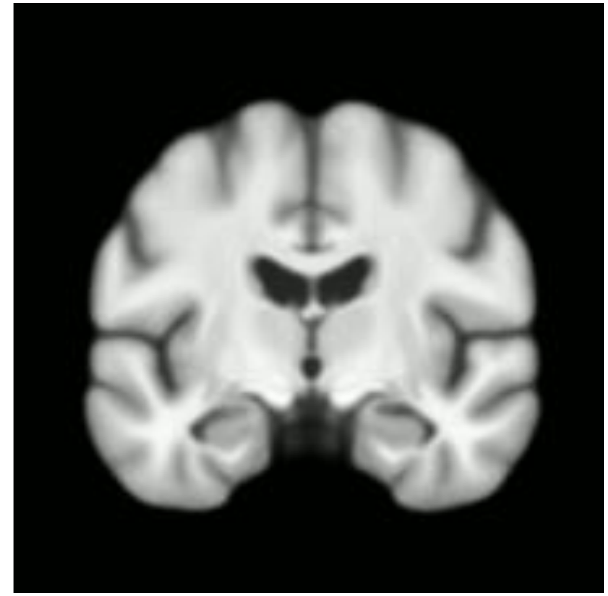
70 AD subjects (ADNI data)



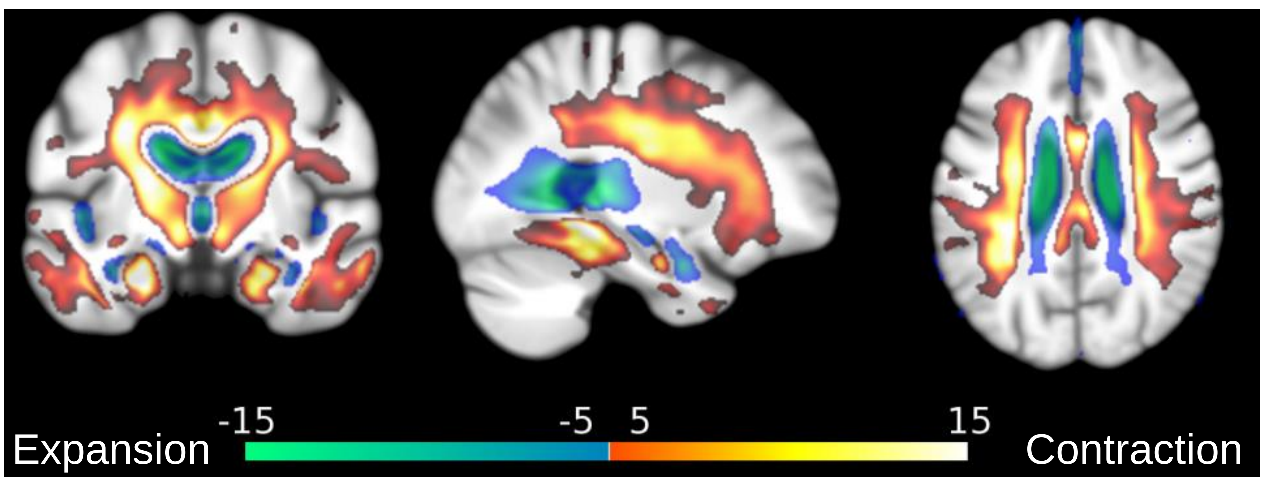
Longitudinal changes in Alzheimer's disease

(141 subjects – ADNI data)



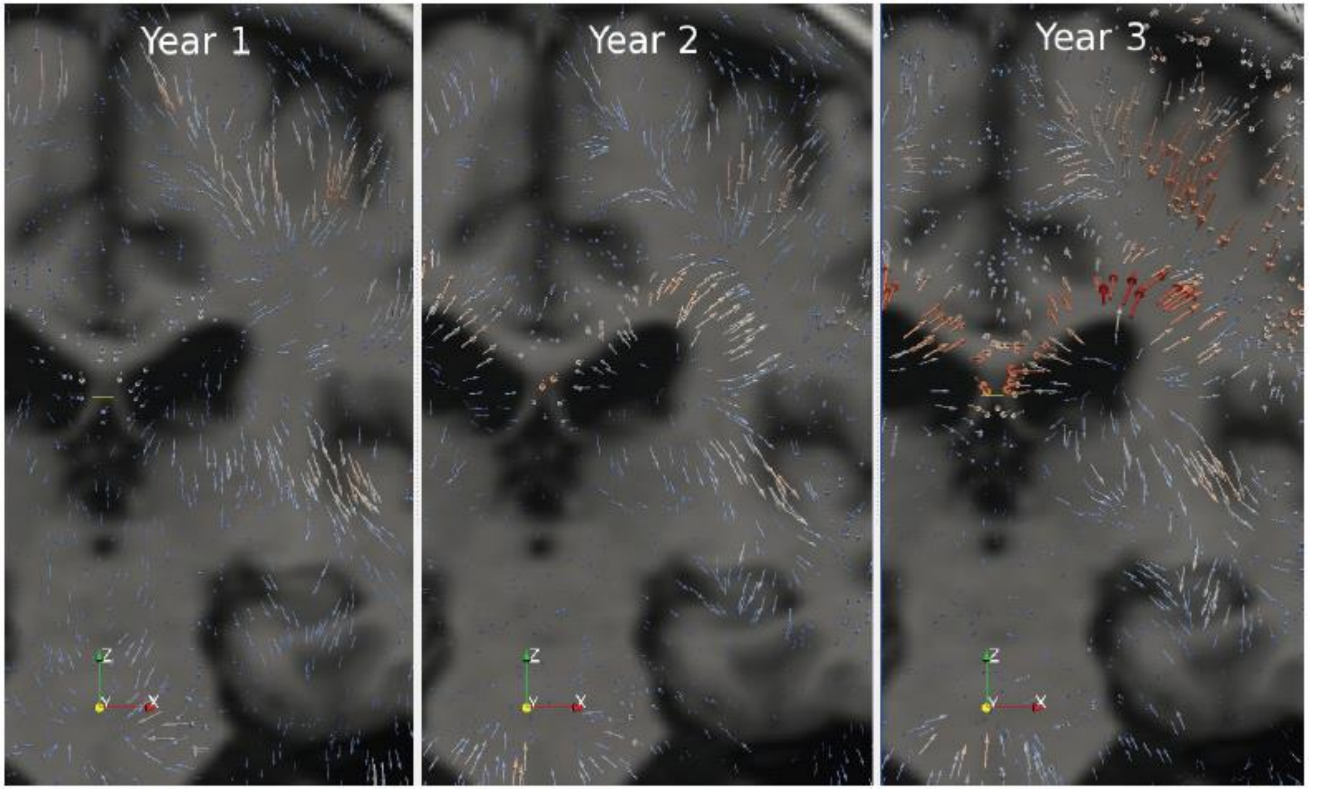


Student's
t statistic



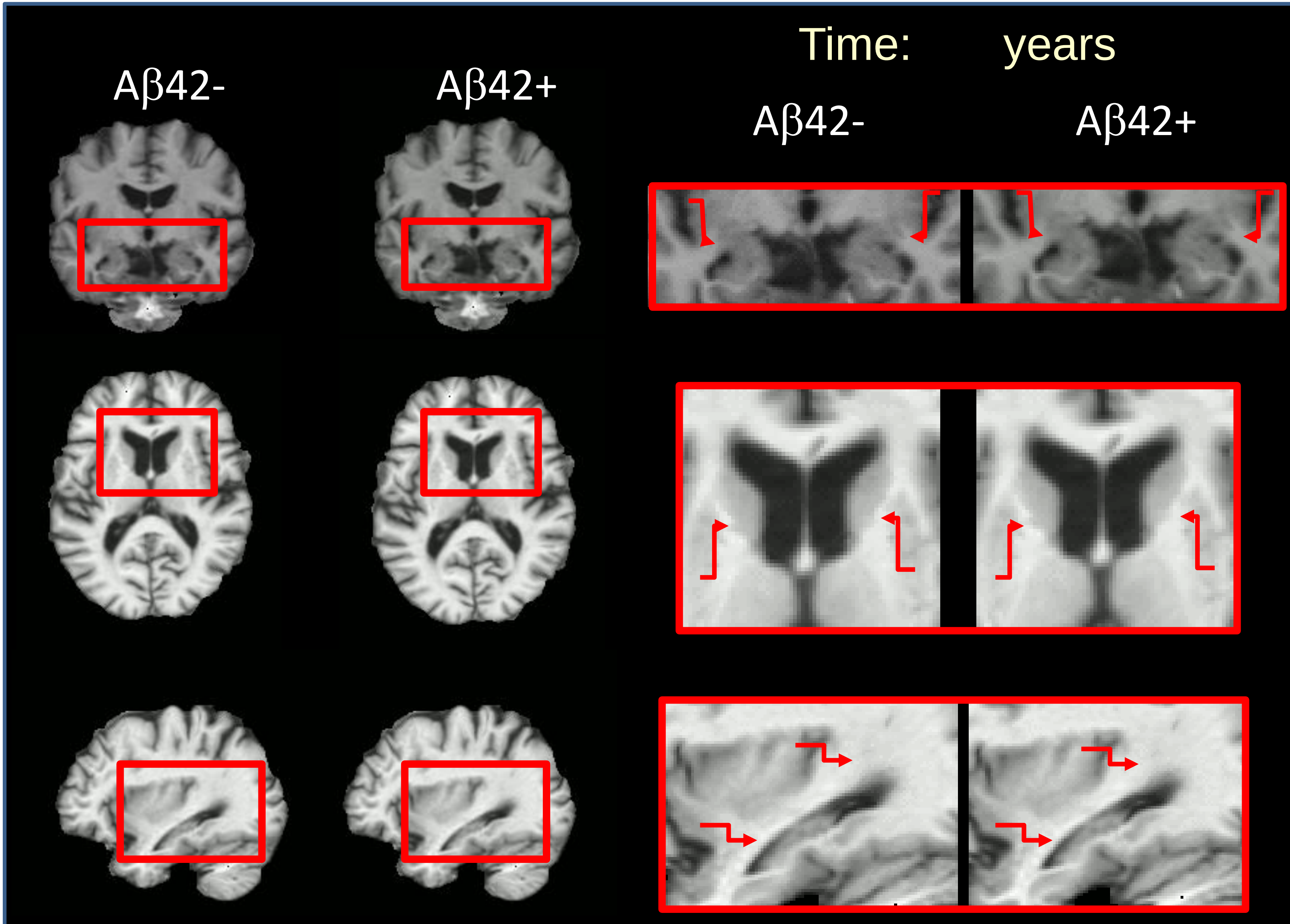
Study of prodromal Alzheimer's disease

- 98 **healthy subjects**, 5 time points (0 to 36 months).
- 41 subjects Aβ42 positive ("at risk" for Alzheimer's)
- **Q: Different morphological evolution for Aβ+ vs Aβ-?**



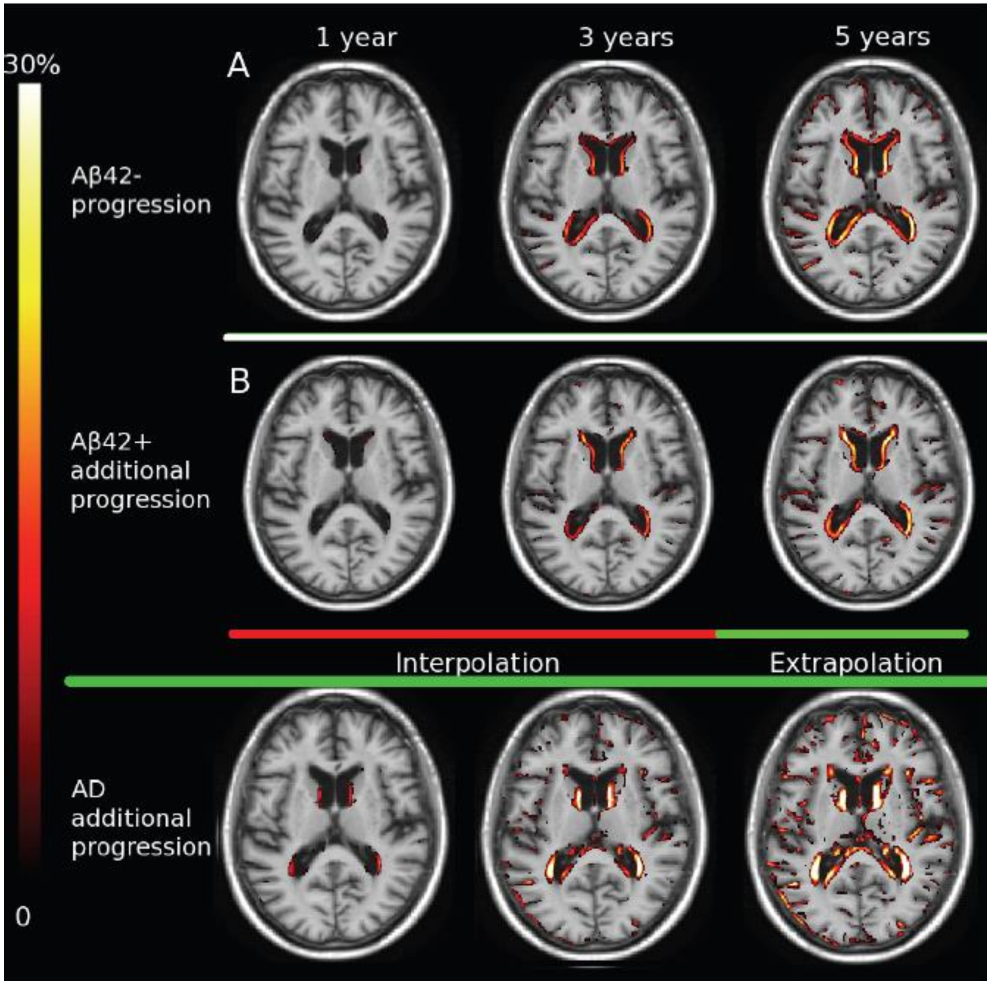
Average SVF for normal evolution (Aβ-)

[Lorenzi, Ayache, Frisoni, Pennec, in Proc. of MICCAI 2011]

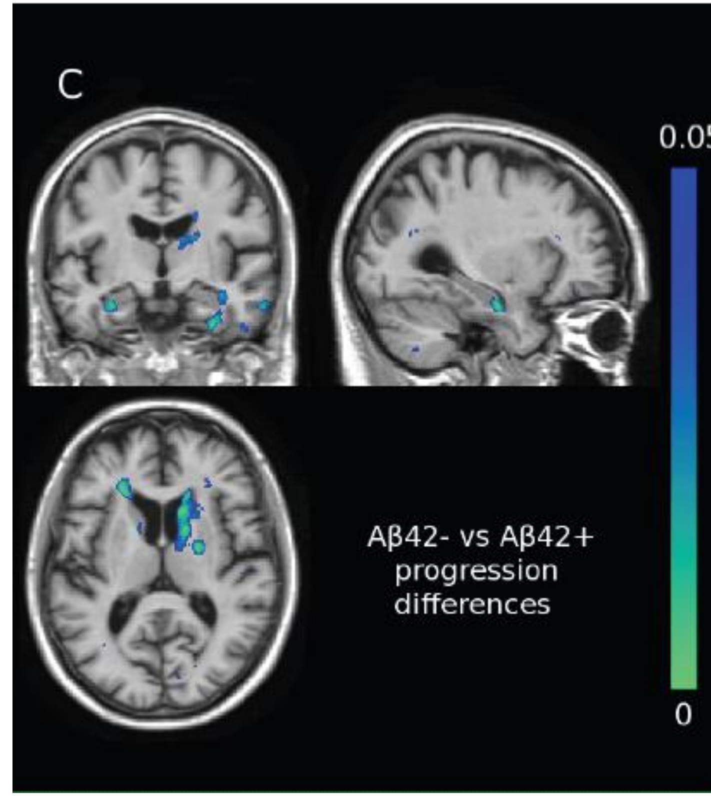

Time: years
Aβ42-
Aβ42+
Aβ42-
Aβ42+

Study of prodromal Alzheimer's disease

Linear regression of the SVF over time: interpolation + prediction



$$T(t) = Exp(\tilde{v}(t)) * T_0$$



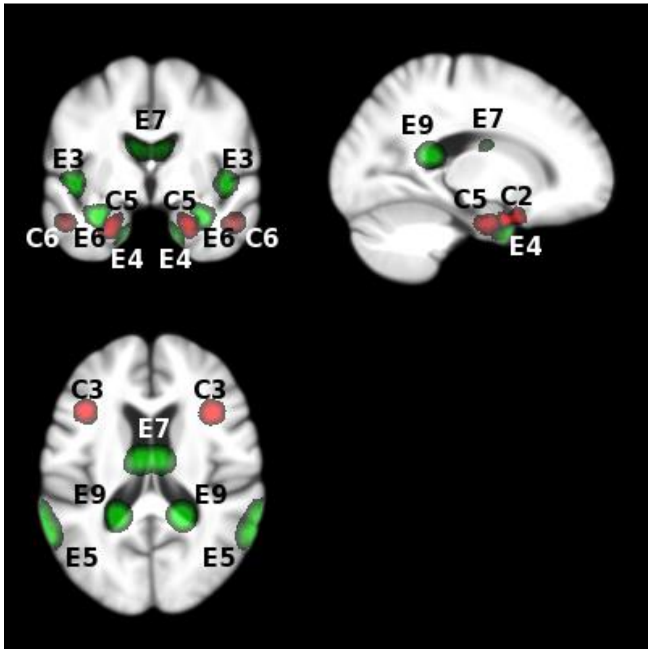
Multivariate group-wise comparison of the transported SVFs shows statistically significant differences (nothing significant on log(det))

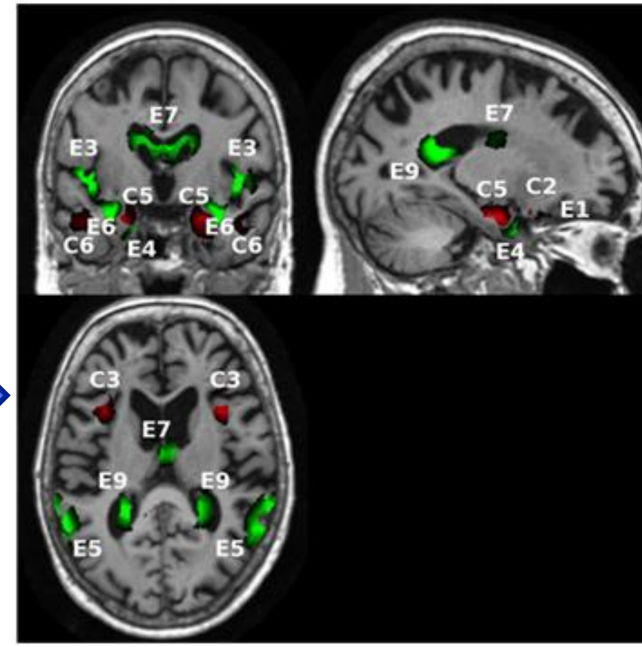
[Lorenzi, Ayache, Frisoni, Pennec, in Proc. of MICCAI 2011]

Group-wise flux analysis in Alzheimer's disease: Quantification

From group-wise…

…to subject specific





sample size ∝ sd/($mean_1$-$mean_2$)

	Regional flux (all regions)	Hippocampal atrophy [Leung 2010] *(Different ADNI subset)*
AD vs controls	164 [106,209]	121 [77, 206]
MCI vs controls	277 [166,555]	545 [296, 1331]

Effect size on left hippocampus

Group	six months	one year	two years
INRIA - Regional Flux	1.02	1.33	1.47

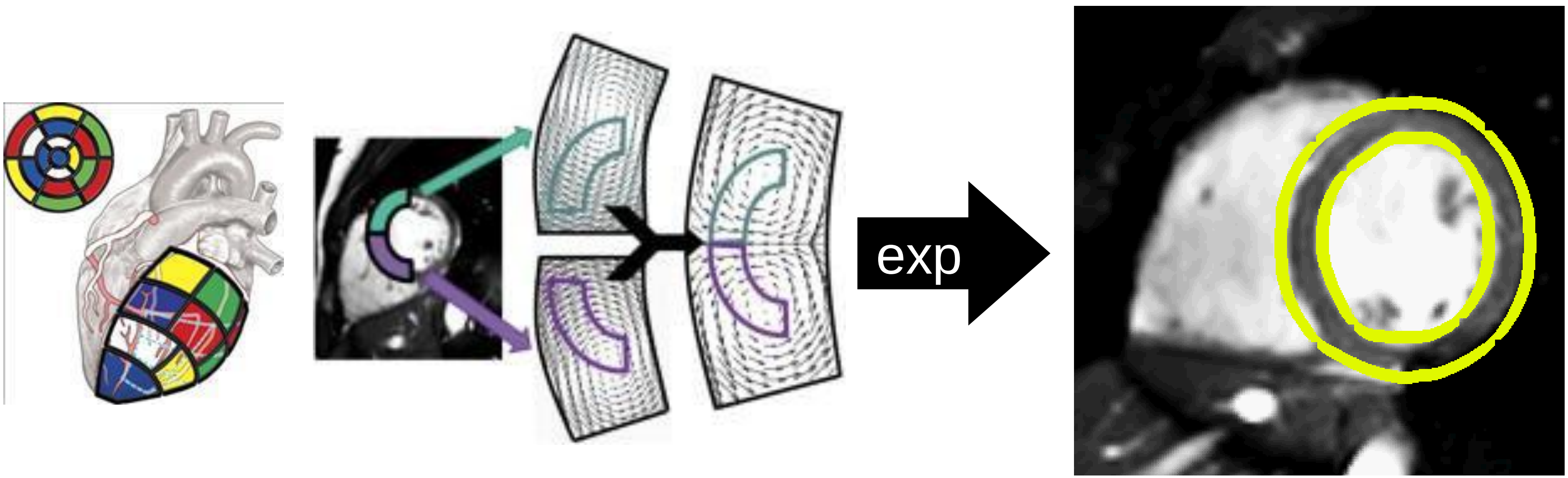
NIBAD'12 Challenge:
Top-ranked on Hippocampal atrophy measures

A powerful framework for statistics

Parametric diffeomorphisms [Arsigny et al., MICCAI 06, JMIV 09]

- One affine transformation per region (polyaffines transformations)
- Cardiac motion tracking for **each subject** [McLeod, Miccai 2013]

Log demons projected but with 204 parameters instead of a few millions



AHA regions — Stationary velocity fields — Diffeomorphism

A powerful framework for statistics

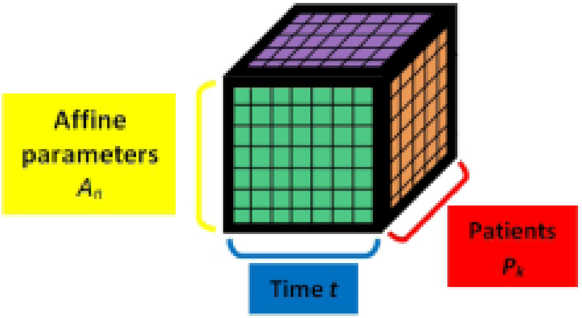
Parametric diffeomorphisms [Arsigny et al., MICCAI 06, JMIV 09]

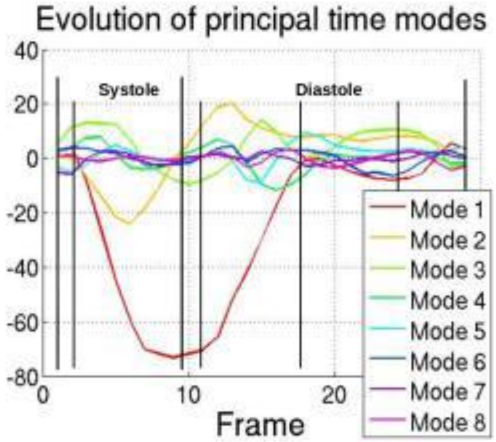
- One affine transformation per region (polyaffines transformations)
- Cardiac motion tracking for **each subject** [McLeod, Miccai 2013]

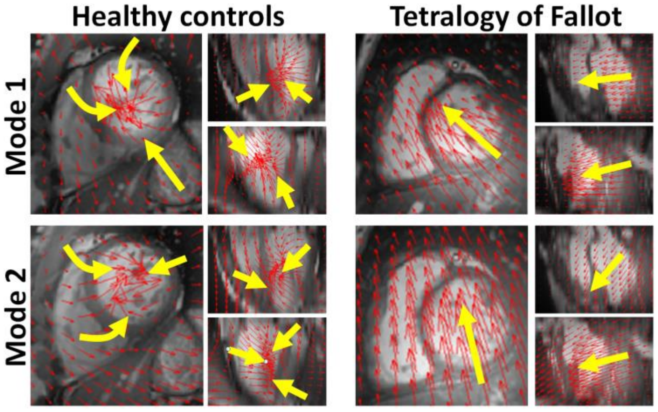
Log demons projected but with 204 parameters instead of a few millions

- **Group analysis** using tensor reduction : reduced model

8 temporal modes x 3 spatial modes = 24 parameters (instead of 204)







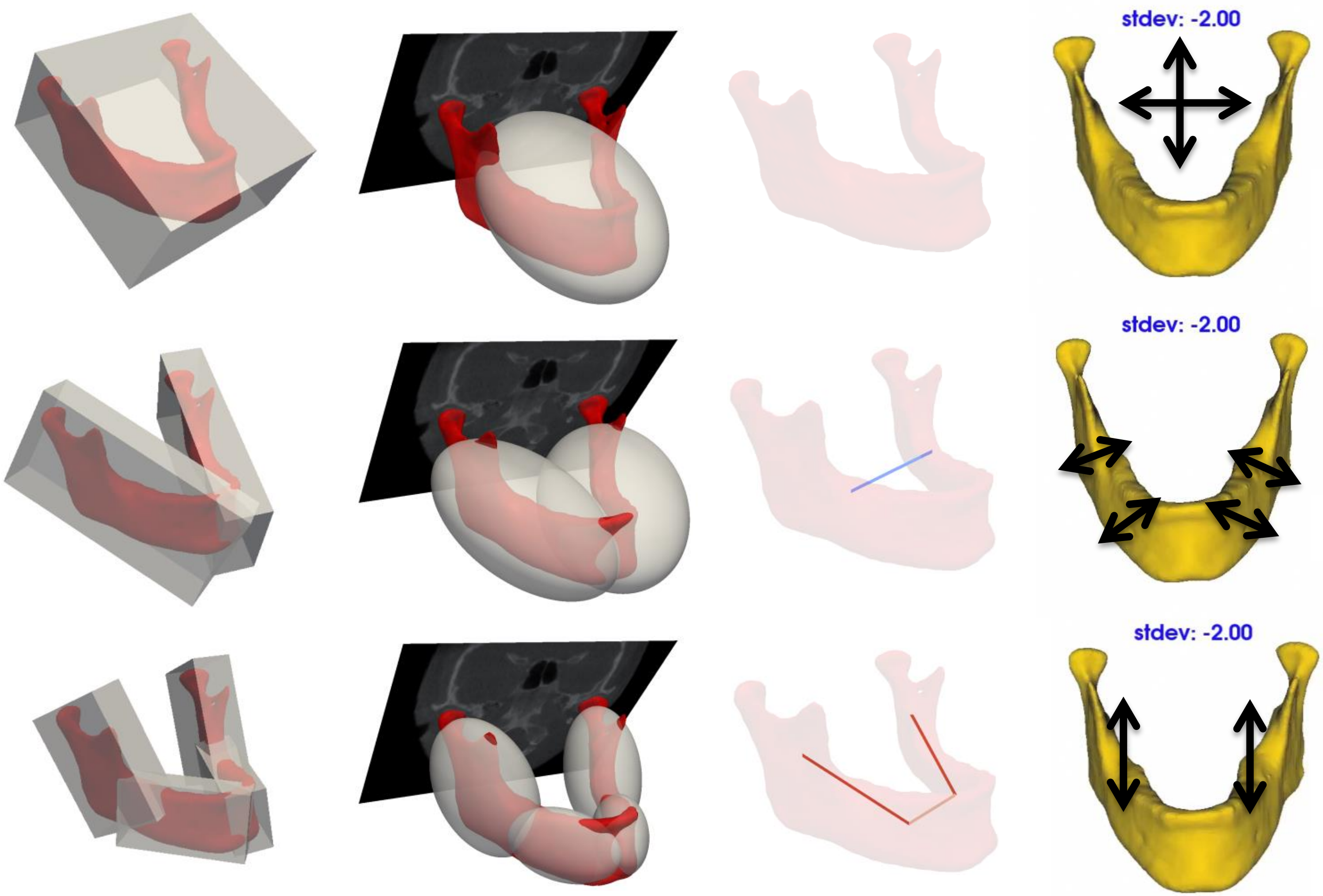
Hierarchical Estimation of the Variability

Oriented bounding boxes | Weights | Structure | First mode of variation

Level 0

Level 1

Level 2



Global scaling

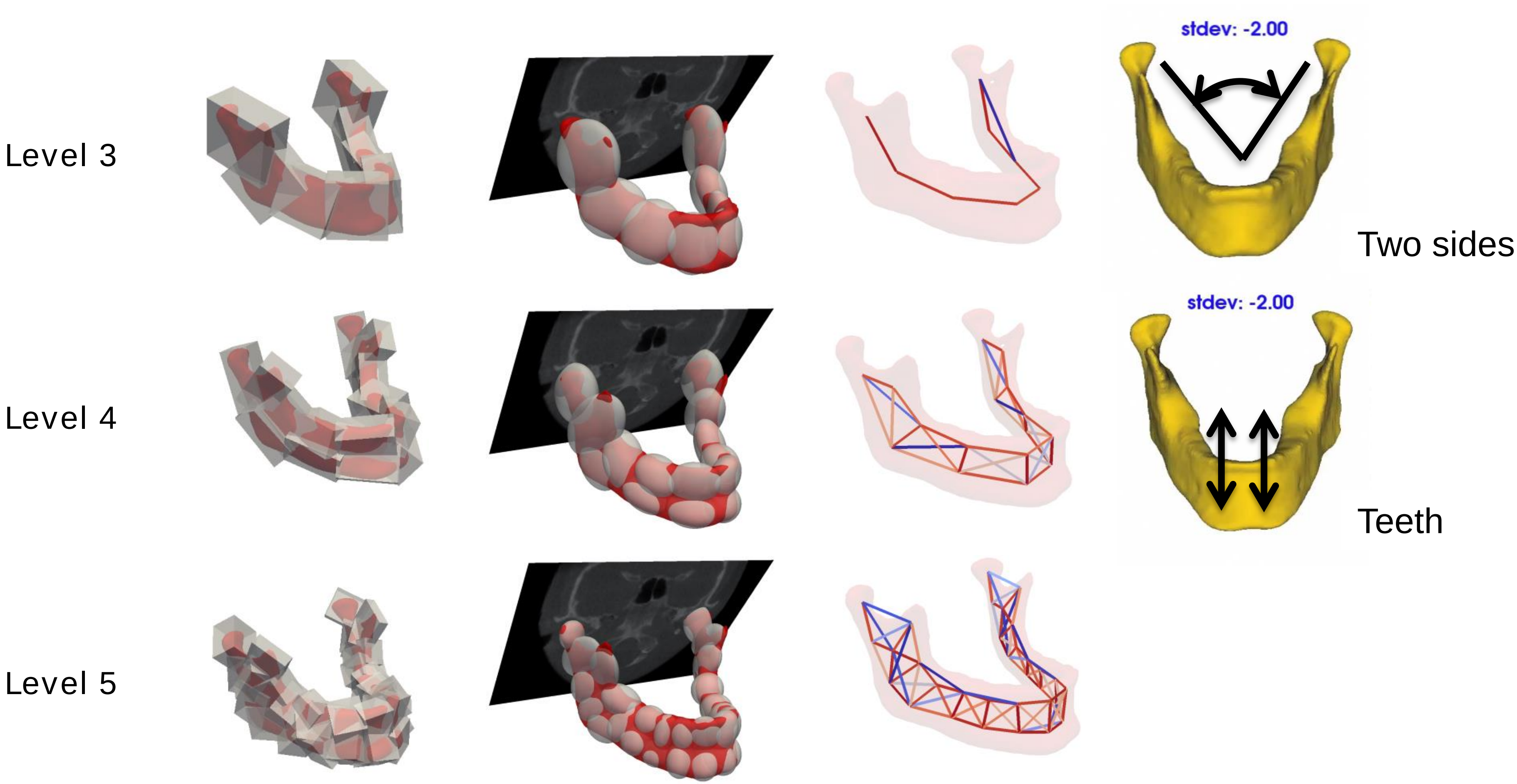
Thickness

Angle and ramus

47 subjects

[Seiler, Pennec, Reyes, Medical Image Analysis 16(7):1371-1384, 2012]

Hierarchical Estimation of the Variability

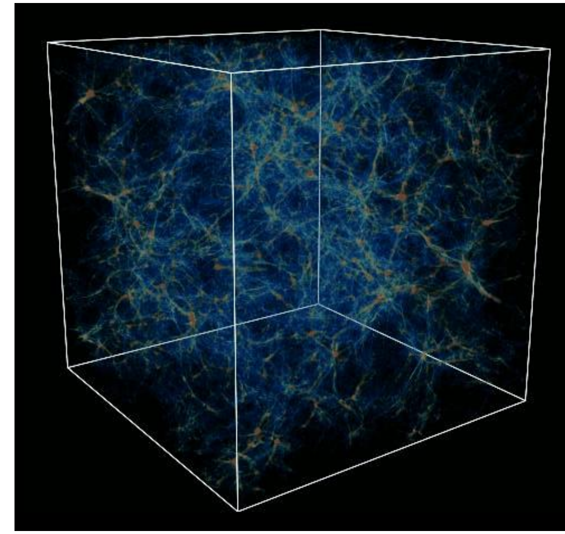


47 subjects **[Seiler, Pennec, Reyes, Medical Image Analysis 16(7):1371-1384, 2012]**

Which space for anatomical shapes?

Physics

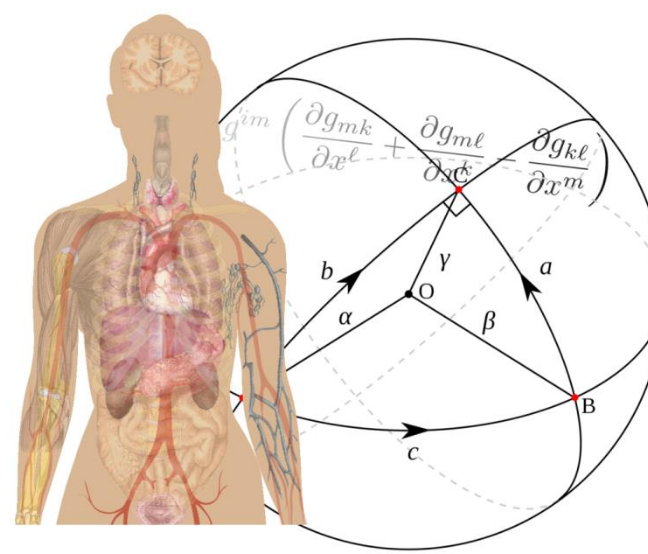
- Homogeneous space-time structure at large scale (universality of physics laws) [Einstein, Weil, Cartan…]
- Heterogeneous structure at finer scales: embedded submanifolds (filaments…)



Modélisation de la structure de l'Univers. NASA

The universe of anatomical shapes?

- Affine, Riemannian of fiber bundle structure?
- Learn locally the topology and metric
  - Very High Dimensional Low Sample size setup
  - Geometric prior might be the key!



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Mathematical Foundations of Computational Anatomy Workshop at MICCAI (last edition in Nagoya 2013)

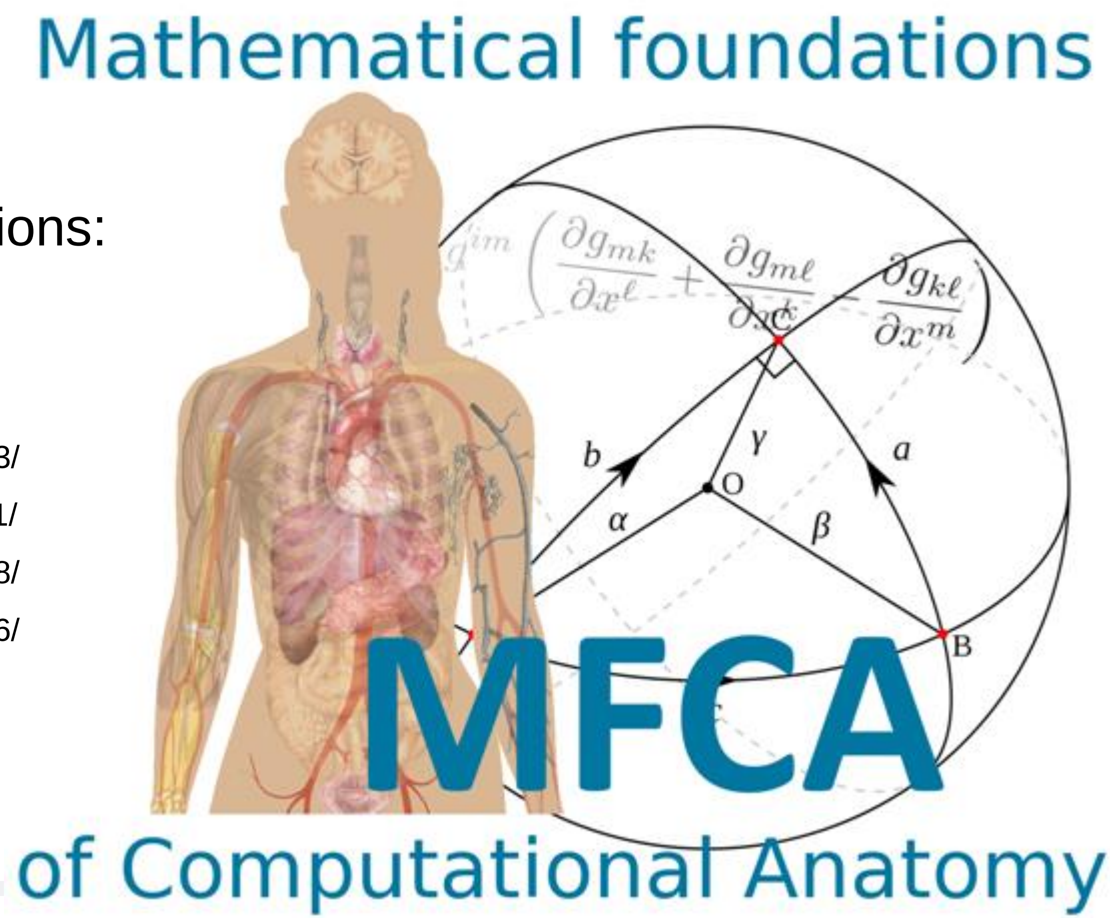
Proceedings of previous editions:
http://hal.inria.fr/MFCA/

http://www-sop.inria.fr/asclepios/events/MFCA13/

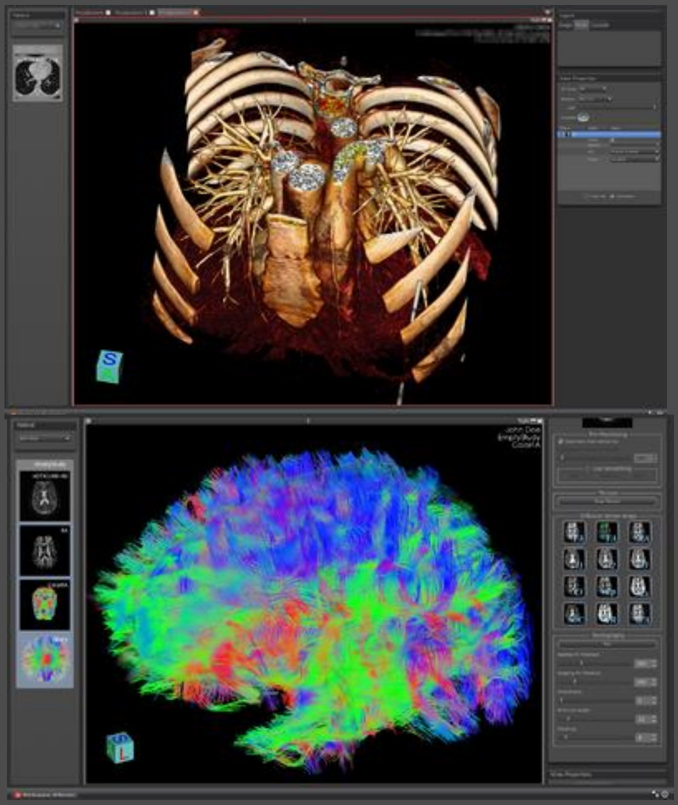
http://www-sop.inria.fr/asclepios/events/MFCA11/

http://www-sop.inria.fr/asclepios/events/MFCA08/

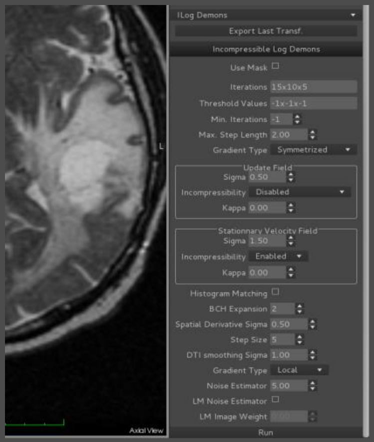
http://www-sop.inria.fr/asclepios/events/MFCA06/



medInria



- Medical image processing and visualization software
- Open-source, BSD license
- Extensible via plugins
- Provides high-level algorithms to end-users
- Ergonomic and reactive user interface



Available registration algorithms :

- Diffeomorphic Demons
- Incompressible Log Demons
- LCC Log Demons

http://med.inria.fr

INRIA teams involved: Asclepios, Athena, Parietal, Visages