Demonology, or a short retrospective of Demons in medical image registration

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On behalf of many people and
the Epidaure / Asclepios team

Asclepios team
2004, route des Lucioles B.P. 93
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http://www-sop.inria.fr/asclepios

Talk overview

The early phase (Thirion)

A Pair and Smooth approach (Cathier)

Adaptive regularization (Stefanescu)

Diffeomorphic demons (Vercauteren)

Extensions and log-demons (Mansi, Yeo, Vercauteren)

The deformable Registration Landscape in 1995

Optical flow

- Horn and Schunck, Artif. Intell. 17, 1981;
- Aggarwal and Nandhakumar, Proc. IEEE 76: 917–935,1988;
- Barron *et al., 1994.*

Linear elastic deformation

- Broit, PhD 1981.
- Bajcsy and Kovacic CVGIP 46, 1989
- Gee, Reivich, Bajcsy, *J. Comp. Assis.Tom. 17,* 1993.

Fluid (images & surface)

- Christensen, Rabbitt, Miller, *Phys. Med. Biol. 39,* 1994.
- Christensen, Rabbitt, Miller.IEEE Trans. Im. Proc. 5(10), 1996.
- Thompson and Toga, IEEE TMI 15(4), 1996.

Mechanical deformations

T is a deformation endoded by its displacement vector field:

$$x_i \mapsto T(x_i) = x_i + u(x_i)$$

Similarity measure is the SSD

$$C = \sum (I(x) - J(x + u(x))^2$$

The differential of this energy is considered as a force:

$$F(x,u) = -(I(x) - J(x+u))\nabla J(x+u) \qquad \textbf{(1)}$$

Mechanical deformations

The force F is applied to the image considered

- Either as a linear elastic material (Lamé Coef.)

$$\mu\nabla^2 u + (\mu+\lambda)\nabla(div(u)) = F \qquad \textbf{(2)}$$

- Or as a viscous fluid (Navier-Stokes, Viscosity Coef.)

$$\mu\nabla^2 v + (\mu+\lambda)\nabla(div(v)) = F \qquad \textbf{(3)}$$

$$\frac{\partial u}{\partial t} = v - (\nabla u)\, v \qquad \textbf{(4)}$$

Equations (2) and (3) are iteratively solved with F computed by (1).
u is computed by integrating equation (4).

Difficulties

- Differential equations are costly to solve
- Regularity of T?
- Small time steps, many iterations
- Very high computation time...

Demons…

Demon

- **Computer Science**

  A program or process that sits idly in the background until it is invoked to perform its task.

- **A person who is part mortal and part god**

  Demigod, deity, divinity, god, immortal - any supernatural being worshipped as **controlling some part of the world** or some aspect of life or who is **the personification of a force**

- **Maxell's demon**

  An imaginary creature who is able to sort hot molecules from cold molecules without expending energy, thus bringing about a general decrease in entropy and violating the second law of thermodynamics.

Demons' algorithm (MRCAS 95, CVPR96, Media98)

Medical Image Analysis (1998) volume 2, number 3, pp 243–260

Image matching as a diffusion process: an analogy with Maxwell's demons

J.-P. Thirion*

INRIA, Equipe Epidaure, 2004 Route des Lucioles BP93, 06902 Sophia-Antipolis, France

Abstract
In this paper, we present the concept of diffusing models to perform image-to-image matching. Having two images to match, the main idea is to consider the objects boundaries in one image as semi-permeable membranes and to let the other image, considered as a deformable grid model, diffuse through these interfaces, by the action of effectors situated within the membranes. We illustrate this concept by an analogy with Maxwell's demons. We show that this concept relates to more traditional ones, based on attraction, with an intermediate step being optical flow techniques. We use the concept of diffusing models to derive three different non-rigid matching algorithms, one using all the intensity levels in the static image, one using only contour points, and a last one operating on already segmented images. Finally, we present results with synthesized deformations and real medical images, with applications to heart motion tracking and three-dimensional inter-patients matching.

Keywords: deformable model, elastic matching, image sequence analysis, inter-patient registration, non-rigid matching

Received October 22, 1996; revised August 8, 1996; March 16, 1998; accepted April 13, 1998

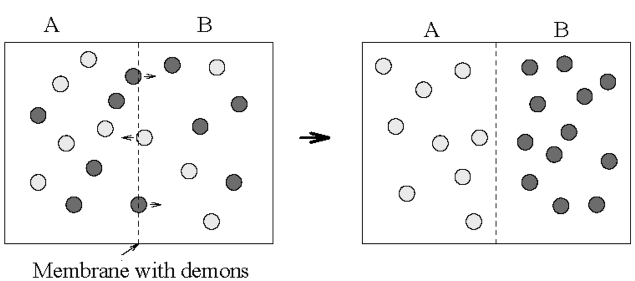


Figure 4. Maxwell's demons and a mixed gas.

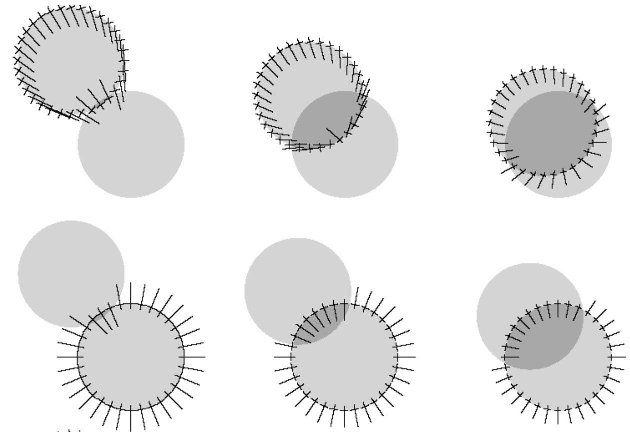
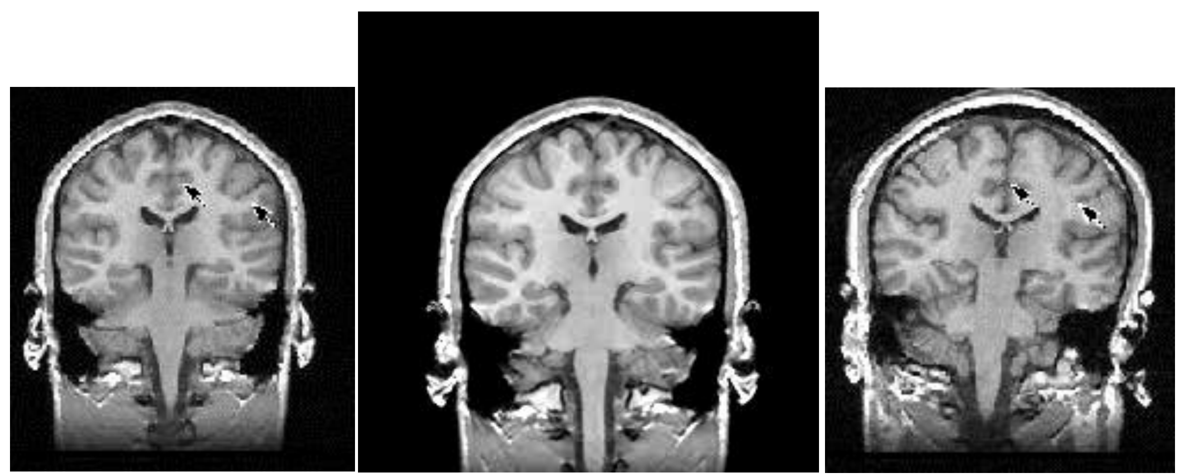


Figure 5. Three iterations of a model based on attraction (top row) and a rigid diffusing model (bottom row). These examples are produced by actual implementations.



Patient 1 Patient 2

Demons' algorithm (MRCAS 95, CVPR96, Media98)

- T_0= Identity
- Correction field $$C_{n+1} = \frac{I - J \circ T_n}{\|\nabla I\|^2 + (I - J \circ T_n)^2} \nabla I$$
- Regularization by Gaussian filtering

Elastic

$$\hat{C}_{n+1} = U_n \circ C_{n+1}$$

$$U_{n+1} = G_\sigma * \hat{C}_{n+1}$$

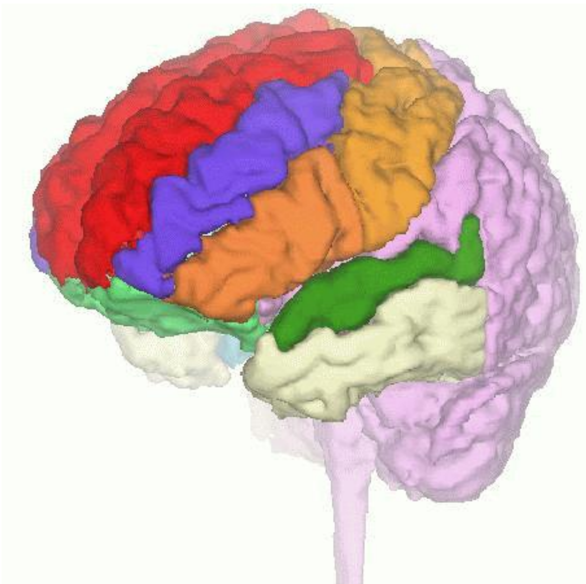
Fluid

$$\tilde{C}_{n+1} = G_\sigma * C_{n+1}$$

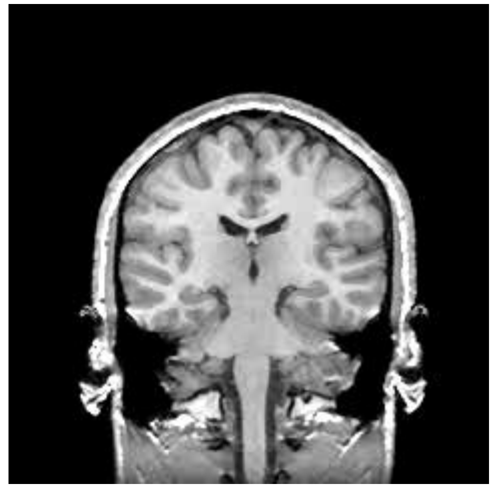
$$U_{n+1} = U_n \circ \tilde{C}_{n+1}$$

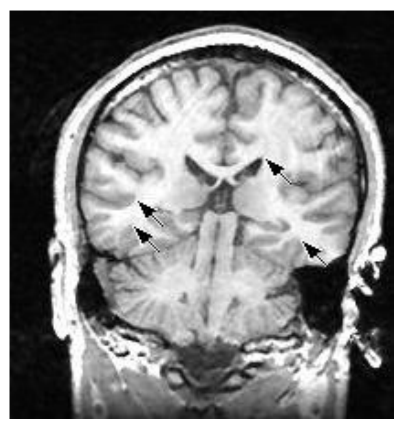
J.P. Thirion: Image Matching as a diffusion process: an analogy with Maxwell's demons. Medical Image Analysis 2(3), 242-260, 1998.

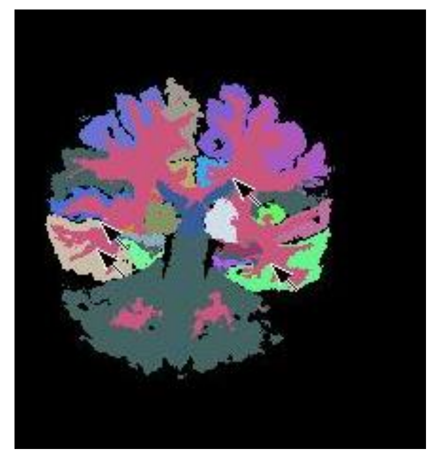
Demons' algorithm (MRCAS 95, CVPR96, Media98)



R. Kikinis
Harvard Medical School







Unbiased Atlases: Guimond 1999

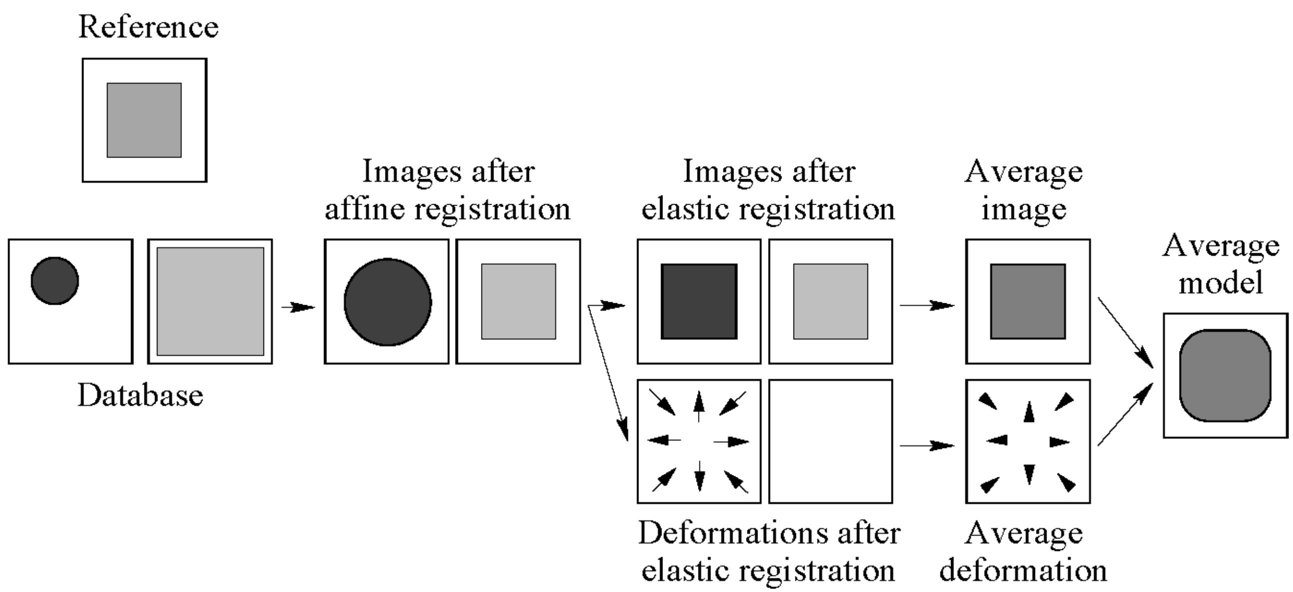


Figure 1: Average model construction method.

Guimond, Meunier, Thirion. Average Brain Models: A Convergence Study. CVIU 77, 1999

- Guimond 2001: VTK implementation (later used for ITK)

Intensity-based deformable registration

Demons algorithm: why does it work?

- + Fast, efficient
- - Do not minimize an energy
  - Difficult to analyze
  - Convergence?
  - Why does that work?
  - How to change the similarity measure?

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Diffeomorphic demons (Vercauteren)

Extensions and log-demons (Mansi, Yeo, Vercauteren)

PASHA: Pair-And-Smooth, Hybrid energy based Algorithm

$$E(C,T) = \frac{1}{\sigma_i^2} SSD(I,J,C) + \frac{1}{\sigma_x^2} \| C - T \|^2 + \mathrm{Reg}(T)$$

- *SSD* : measures the similarity of intensities
- *Reg* : regularization energy (quadratic)
- σ_x , σ_i : smoothing and noise parameters
- *C* : correspondences between points (vectors field)
- *T* : transformation (regularized vector field)

- Correspondences (matches) as an auxiliary variable

P. Cachier E. Bardinet, E. Dormont, X. Pennec and N. A.: *Iconic Feature Based Nonrigid Registration: the PASHA Algorithm*, Comp. Vision and Image Understanding (CVIU), Special Issue on Non Rigid Registration, 89 (2-3), 272-298, 2003.

PASHA: Pair-And-Smooth, Hybrid energy based Algorithm

$$E(C,T) = \frac{1}{\sigma_i^2} SSD(I,J,C) + \frac{1}{\sigma_x^2} \| C - T \|^2 + \mathrm{Reg}(T)$$

Alternated minimization

- Minimization with respect to *C* :
  - Find matches between points by optimizing E_S + **in the neighborhood of** *T*
  - Gradient descent (1^{st}, 2^{bd} order, e.g. Gauss-Newton)
- Minimization with respect to *T* :
  - Find a smooth transformation that approximates *C*
  - Quadratic energy $\Rightarrow$ convolution
- **Interest**: fast computation

Gauss-Newton optimization of the correspondences

$$E(C) = \int (I(x) - J(C(x))^2 .dx + \frac{\sigma_i^2}{\sigma_x^2} \int \| C(x) - T(x) \|^2 .dx$$

Newton optimization

- Second order Taylor expansion of E(C)
- Hessian matrix can be null or negative

Gauss-Newton

- 1st order Taylor expansion of error

$$\left[I\text{-}J \circ (T+u)(x) \right] = \left[I - J \circ T\,(x) \right] + (\nabla J \circ T\,)^T .u(x) + O(||u(x)||^2)$$

- Solve approximated SSD Criterion around C=T

$$\mathrm{E(C+u)} \approx SSD(T) + 2 \int (J \circ T - I).(\nabla J \circ T)^t .u$$
$$+ \int u^t .(\nabla J \circ T).(\nabla J \circ T)^t .u + 2 \frac{\sigma_i^2}{\sigma_x^2} \int (C - T)^t .u + \frac{\sigma_i^2}{\sigma_x^2} \|u\|^2$$

Gauss-Newton optimization of the correspondences

$$E(C) = \int (I(x) - J(C(x))^2 .dx + \frac{\sigma_i^2}{\sigma_x^2} \int \| C(x) - T(x) \|^2 .dx$$

Exact solution of the quadratic approximation of the SSD

- Solve $\left[(\nabla J \circ T).(\nabla J \circ T)^t + \frac{\sigma_i^2}{\sigma_x^2} Id \right] .u = (J \circ T - I).(\nabla J \circ T)$
- By inversion lemma: $u = \frac{(J \circ T - I).(\nabla J \circ T)}{\| \nabla J \circ T \|^2 + \sigma_i^2 / \sigma_x^2}$
- Local estimation of intensity variance: $\sigma_i^2 = (J \circ T - I)^2$
- Assuming isotropic voxel size: $\sigma_x^2 \approx 1$

$$u = \frac{I - J \circ T}{\| \nabla I \|^2 + (I - J \circ T)^2} \nabla I$$

Important Practical Remark

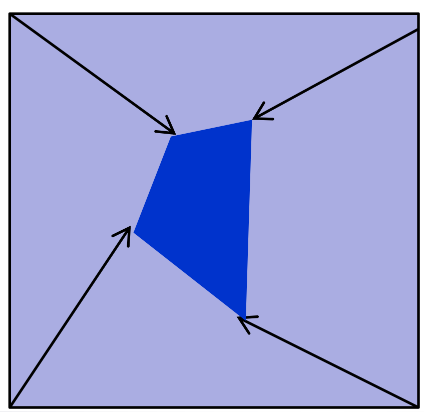
$$u = \frac{I - J \circ T}{\|\nabla I\|^2 + (I - J \circ T)^2} \nabla I$$

- Norm of update is bounded by construction

$$\left(\|\nabla I\| - (I - J \circ T)\right)^2 = \|\nabla I\|^2 + (I - J \circ T)^2 - 2(I - J \circ T)\|\nabla I\| > 0$$

$$\|u\| \leq 1/2$$

- Update is diffeomorphic by tri-linear interpolation!



Efficient Regularization

Quadratic regularizer $\mathrm{Reg}(T) = \int \sum_{k=1}^{\infty} \frac{\sum_{i_1 \ldots i_k} \left\| \partial_{i_1} \ldots \partial_{i_k} (T - Id) \right\|^2}{\sigma_d^{2k} . k!}$

Euler Lagrange optimization of $\mathrm{E(T)} = \int \left\| \mathrm{C - T} \right\|^2 + \mathrm{Reg}(T)$

$$C - T + \sum_{k=1}^{\infty} \frac{(-1)^k \Delta^k (T - Id)}{\sigma_d^{2k} . k!} = 0$$

Solution: Gaussian smooting $\mathrm{T_{opt}} = G_\sigma * C \quad \text{with} \quad \sigma = 1/\sigma_d$

- Pennec, Cachier, Ayache. Understanding the ``Demon's Algorithm'': 3D Non-Rigid registration by Gradient Descent. MICCAI 1999.

Extension to a family of quadratic filters

$$G_{\sigma,\kappa}(\mathbf{u}) = \frac{1}{(\sigma\sqrt{2\pi})^3(1+\kappa)} \left(\mathrm{Id} + \frac{\kappa}{\sigma^2} \mathbf{u}\mathbf{u}^T \right) \exp\left(\frac{\mathbf{u}^T\mathbf{u}}{2\sigma^2} \right)$$

- P. Cachier and N. Ayache. Isotropic energies, filters and splines for vectorial regularization. J. of Math. Imaging and Vision, 20(3):251-265, May 2004.

Mixed Elastic / Fluid Regularization

$$E(C_n, T_n) = E_S(I, J, C_n) + \sigma \| C_n - T_n \|^2$$

$$+ \sigma\lambda.\mathrm{Reg}(T_n) + \sigma\lambda\left[\omega.\mathrm{Reg}(T_n - T_{n-1}) + (1-\omega)\mathrm{Reg}(T_n)\right]$$

- Result is still obtained by convolution:

$$T_n = (1-\omega).\ K * C_n + \omega.(T_n + K * (C_n - T_{n-1}))$$

- **Advantages**:
  - Mixes fluid and elastic
  - handles large displacements

P. Cachier N. A., *Isotropic Energies, Filters and Splines for Vector Field Regulatization*, J. of Mathematical Imaging and Vision, 20: 251-265, 2004

The Demons/PASHA Framework

Efficient energy minimization

$$E(C,T,\dot{T}) = \boxed{E_S(I,J,C)} + \boxed{\sigma \int \| C - T \|^2} + \boxed{\lambda \,\mathrm{Reg}(T) + \mu \,\mathrm{Reg}(\dot{T})}$$

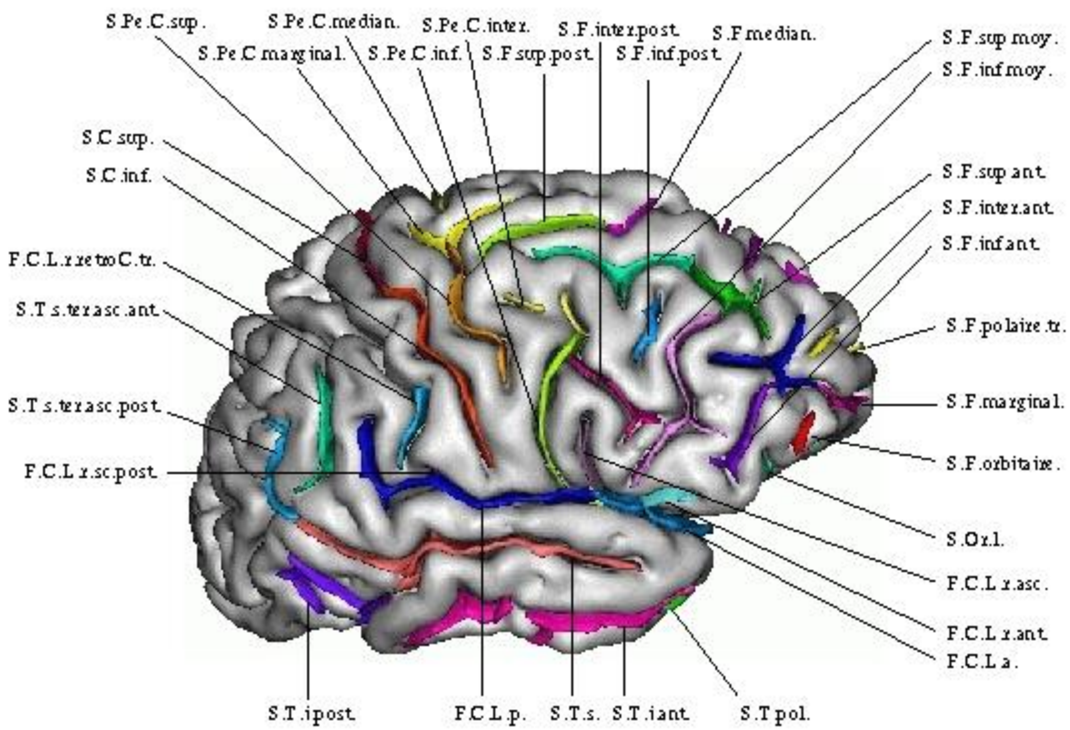
similarity *Auxiliary* *Elastic + Fluid Regularity*

Alternate Minimization

- on C, Correspondance Field (image forces)
  Gauss-Newton gradient descent: normalized optical flow
- on T, Deformation Field (regularization)
  Gaussian convolution

•P. Cachier E. Bardinet, E. Dormont, X. Pennec and N. A.: Iconic Feature Based Nonrigid Registration: the PASHA Algorithm, Comp. Vision and Image Understanding (CVIU), 89 (2-3), 272-298, 2003.

Features - Intensity -Semantics



JF. Mangin, D. Rivière, SHFJ-CEA

ARC BrainVar: CEA-Asclepios--Salpêtrière-Visages

Inter-subject registration

Add geometric constraints

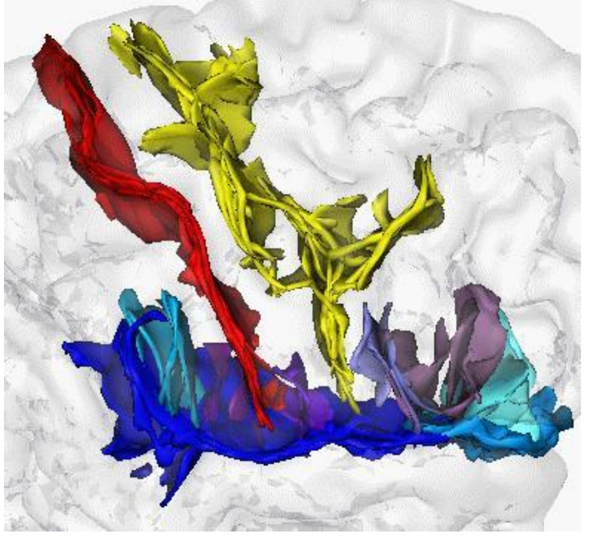
- Correspondences C_2 between sulci
- Registration energy becomes

$$E(C_1, C_2, T) = S(I, J, C_1) + \sigma.\| C_1 - T \|^2$$
$$\boxed{+\sigma.\gamma.\| C_2 - T \|^2} + \sigma.\lambda.\mathrm{Reg}(T)$$

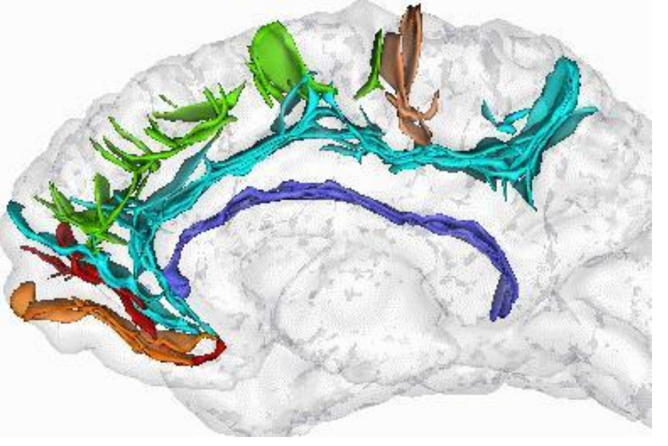
- Algorithm in 3 steps:
  - Min. w.r.t. C_1 by gradient descent
  - Min. w.r.t. C_2 by nearest neighbor search
  - Min. w.r.t. T : explicit solution (convolution + spline)

[P. Cachier et al, MICCAI 2001]

Results with 5 subjects



Intensity + Features



Intensity + Features

P. Cachier, J.-F. Mangin, X. Pennec, D. Rivière, D. Papadopoulos, J. Régis, N. A. Multisubject Non-Rigid Registration of Brain MRI using Intensity and Geometric Features. MICCAI'01, LNCS vol 2208, 734-742, 2001.

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Towards more functional registration algorithms (PhD Radu Stefanescu, 2002-2005)

- Adapt regularization with respect to the tissues
  - Non stationary smoothing simulating elastic/fluid
- Correspondences are fuzzy or less reliable at certain places
  - Pathologies, homogeneous intensity areas
  - Register only certain areas, interpolate the remaining
  - Choice of interest points: selective registration
- Fast parallel resolution (1-5 min)
  - High Performance Computing: PC cluster

Revisiting Regularization

$$E(C,T,\dot{T}) = E_S(I,J,C) + \sigma \int \| C - T \|^2$$

$$+ \lambda \int \| \nabla (T - Id) \|^2 + \mu \int \left\| \nabla \dot{U} \right\|^2$$

Modulate regularization as a function of

- 1- local variability (statistics on anatomy)
- 2- local information (presence of texture/edges)

R. Stefanescu, X. Pennec , N. A., *Grid Powered Nonlinear Image Registration with Locally Adaptive Regularization*, Medical Image Analysis, Sept 2004 (also MICCAI'03)

Inhomogeneous Regularization Implementation

$$E(C,T,\dot{T}) = E_S(I,J,C) + \sigma \int \| C - T \|^2$$

$$+ \int \lambda . \| \nabla (T - Id) \|^2 + \int \mu . \left\| \nabla \dot{U} \right\|^2$$

Modulate regularization into non-stationary heat equation

- No more Gaussian smoothing
- Use 1st order gradient descent

R. Stefanescu, X. Pennec , N. A., *Grid Powered Nonlinear Image Registration with Locally Adaptive Regularization*, Medical Image Analysis, Sept 2004 (also MICCAI'03)

- Coupled PDEs with Gaussian convolutions
  - Cahill, Noble, Hawkes, MICCAI 2009

Non Stationary Elastic Regularization

$$\frac{\partial T}{\partial t} = div(D\nabla(T - Id))$$

Diffusion or stiffness tensor

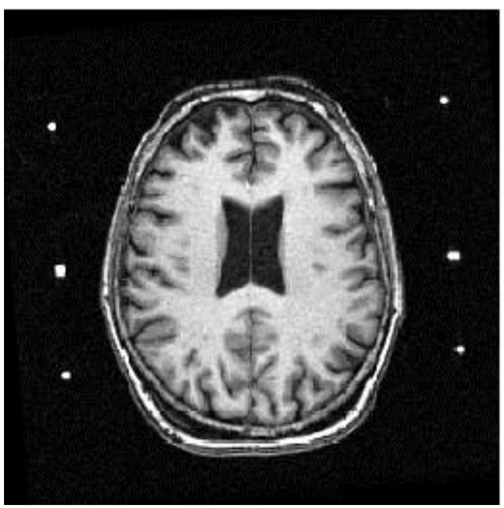
- Encodes a priori variability
- Image and application dependent
- Scalar or tensor (directional)

Non Stationary Elastic Regularization

$$\frac{\partial T}{\partial t} = div(D\nabla(T - Id))$$

Diffusion or stiffness tensor

Source image

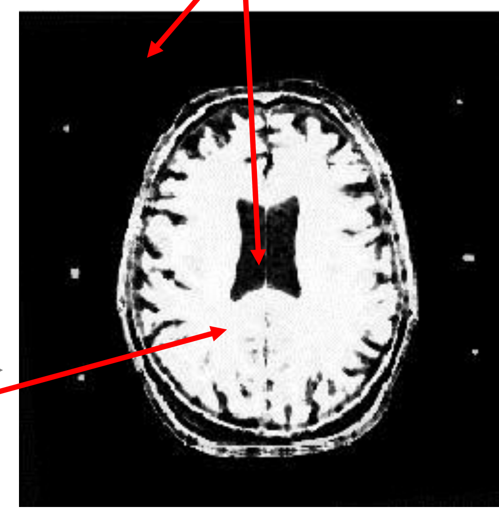


Inter-subject brain registration:

$$D = P(grey) + P(white)$$

0,01

0,9



Non Stationary Fluid Regularization

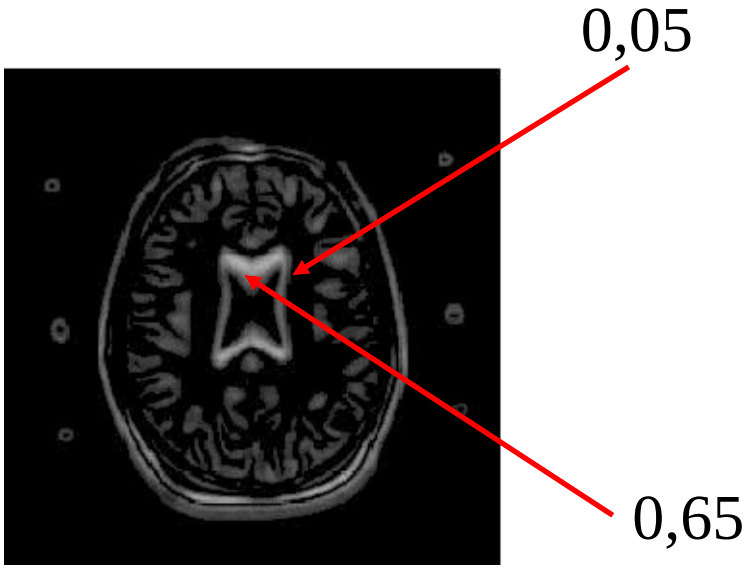
Inspired from non-stationary image diffusion

- Weickert 1997, 2000
- Solved using AOS scheme

$$\frac{\partial u_i}{\partial t} = (1-k)\Delta u_i$$

Confidence in the correction field

- k ~ 1 for edges (driving forces)
- k ~ 0 for uniform regions (interpolation)



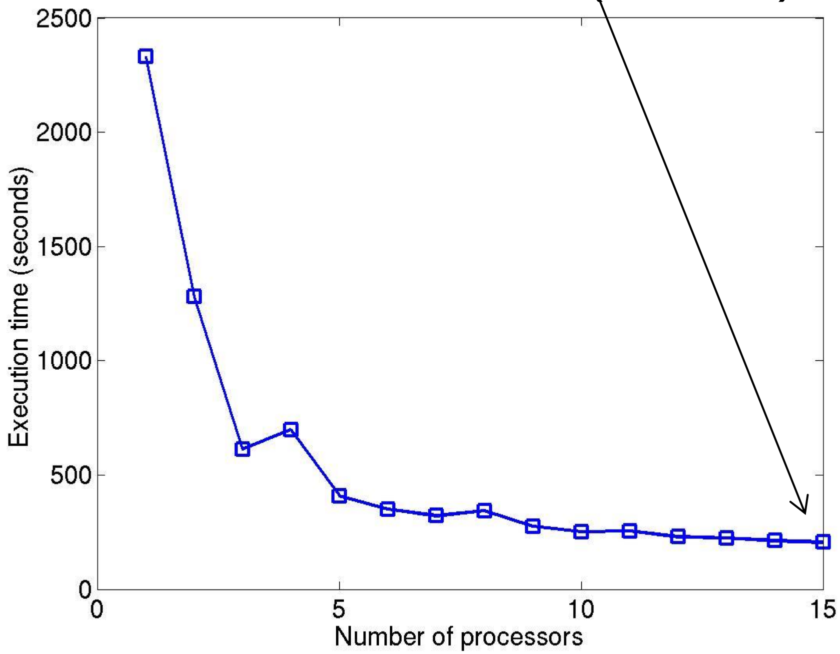
- Used to model pathologies (e.g. tumors)

Performance issues: no closed-form solution!

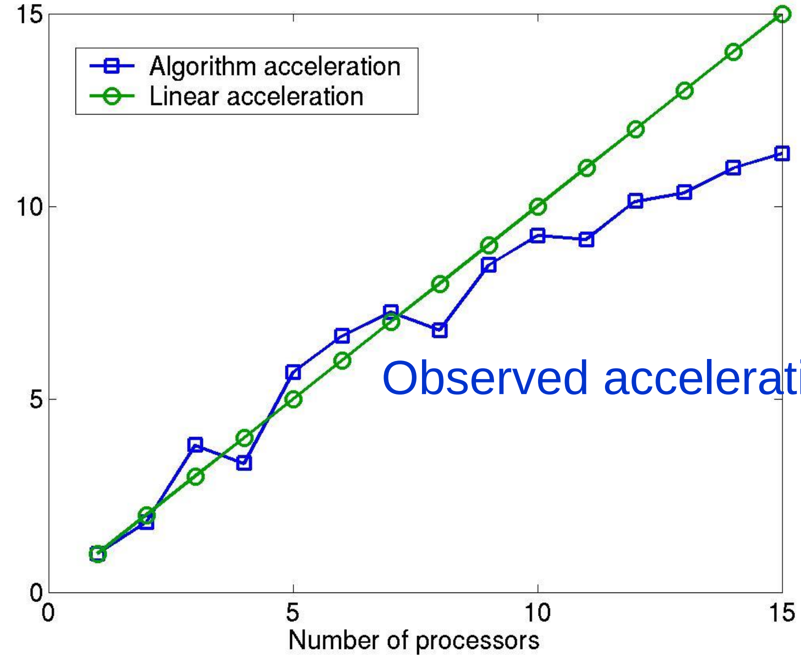
Parallel implementation

- Semi-implicit AOS scheme
- Parallelization using Thomas algorithm

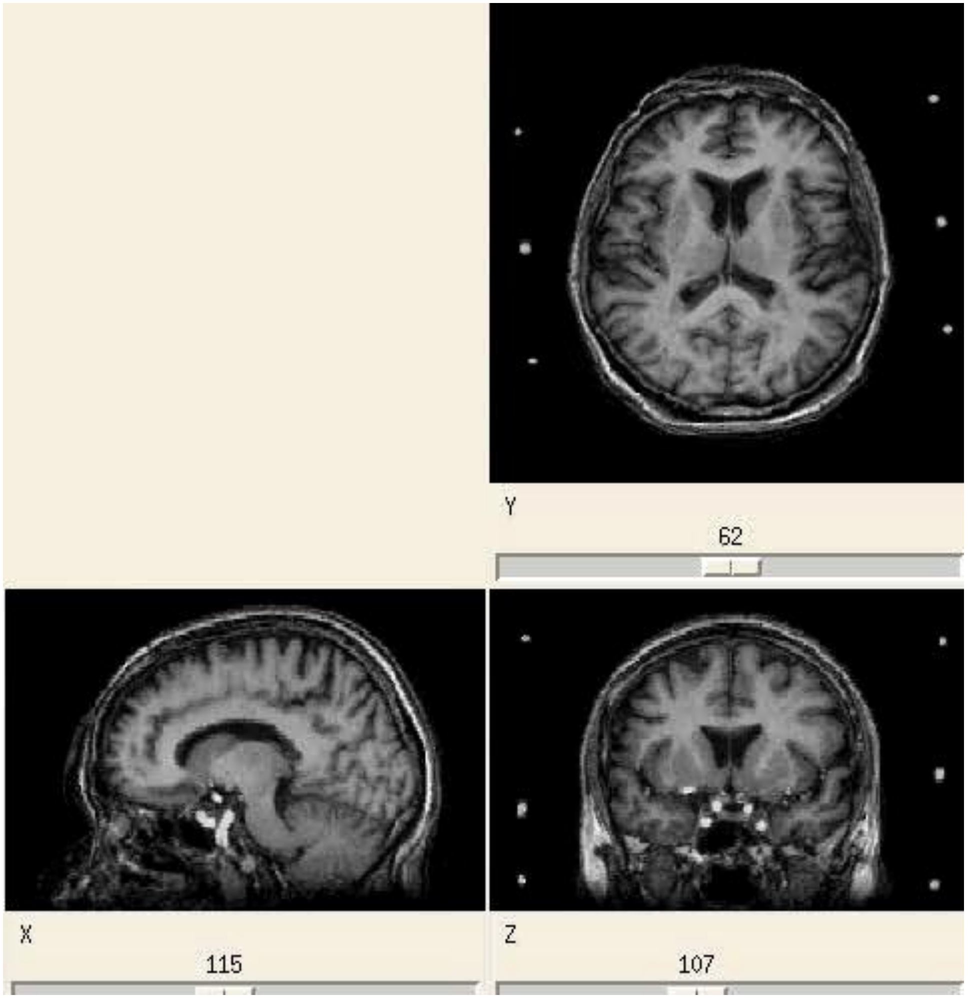
Images 256 x 256 x 60 :
3 minutes 30 (in 2005)



Ideal linear acceleration



Inter-subject registration
Affine transformation



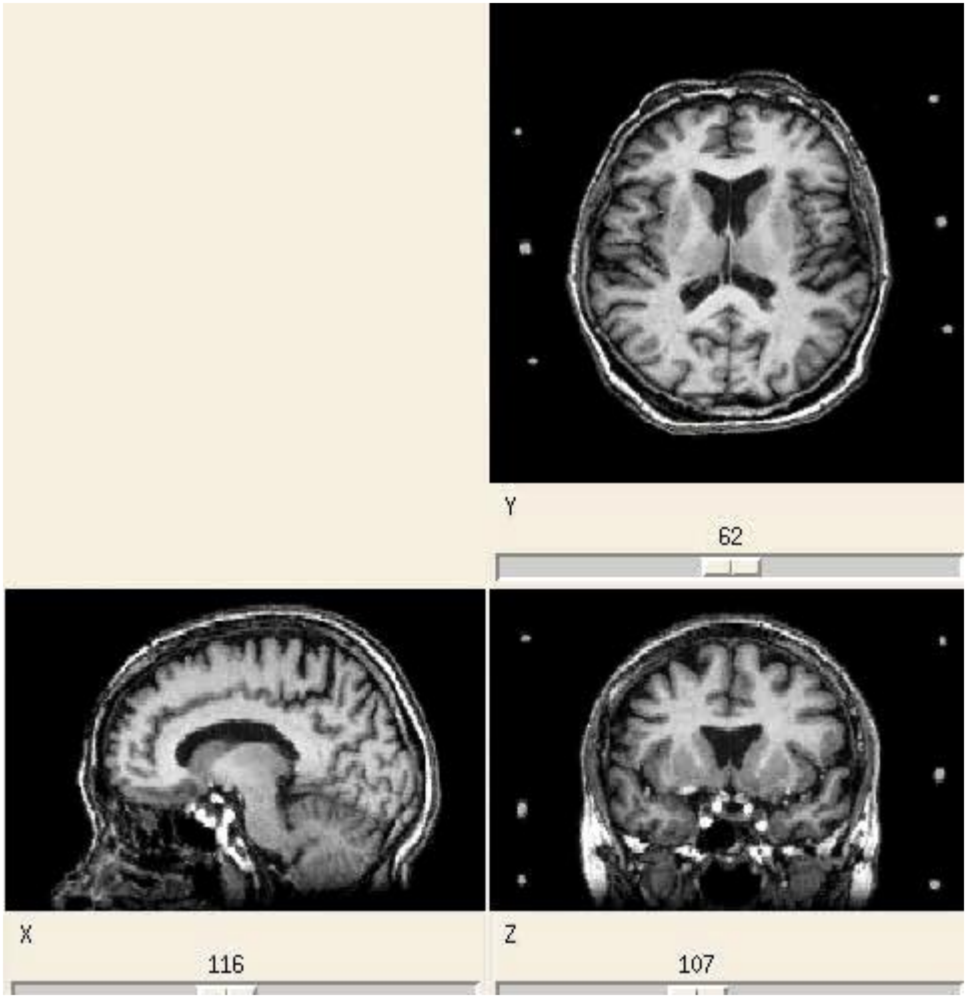
MR T1 Images

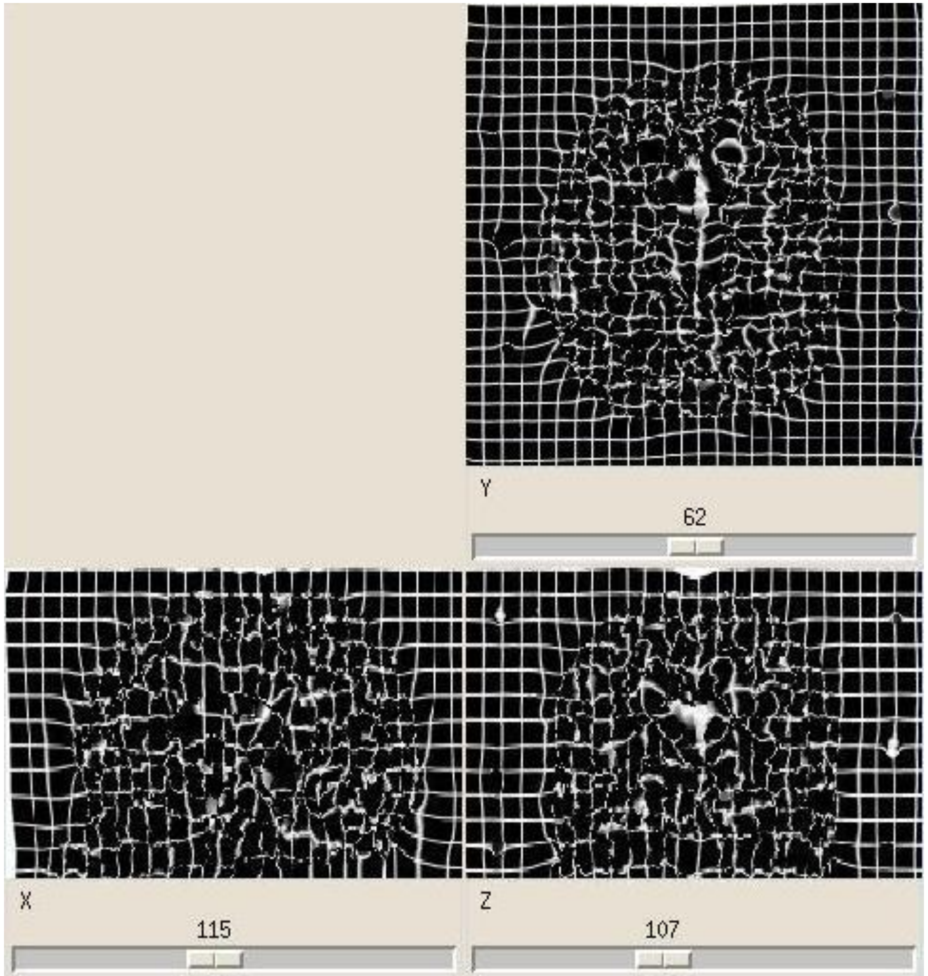
256x256x120 voxels

Atlas to patient registration for radiotherapy planning

Correct size and position but high remaining variability in cortex and deep structures

Inter-subject registration
Fluid regularization



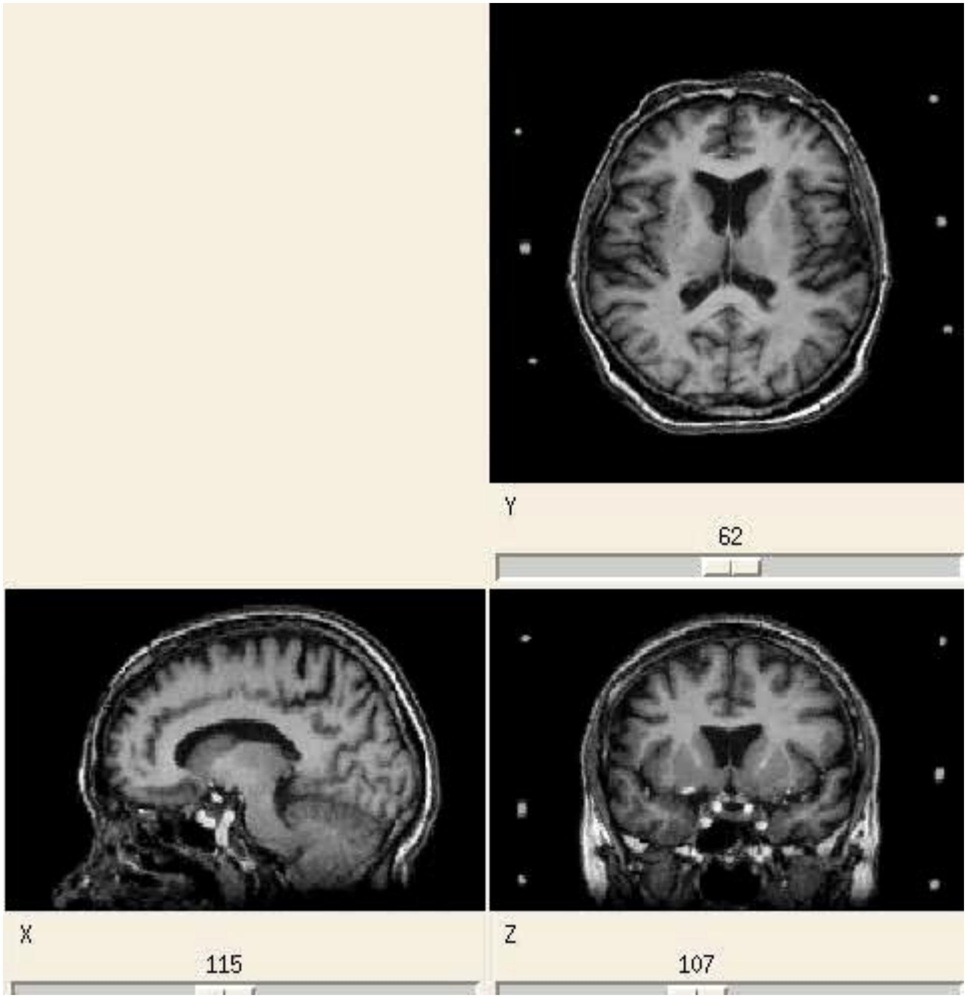


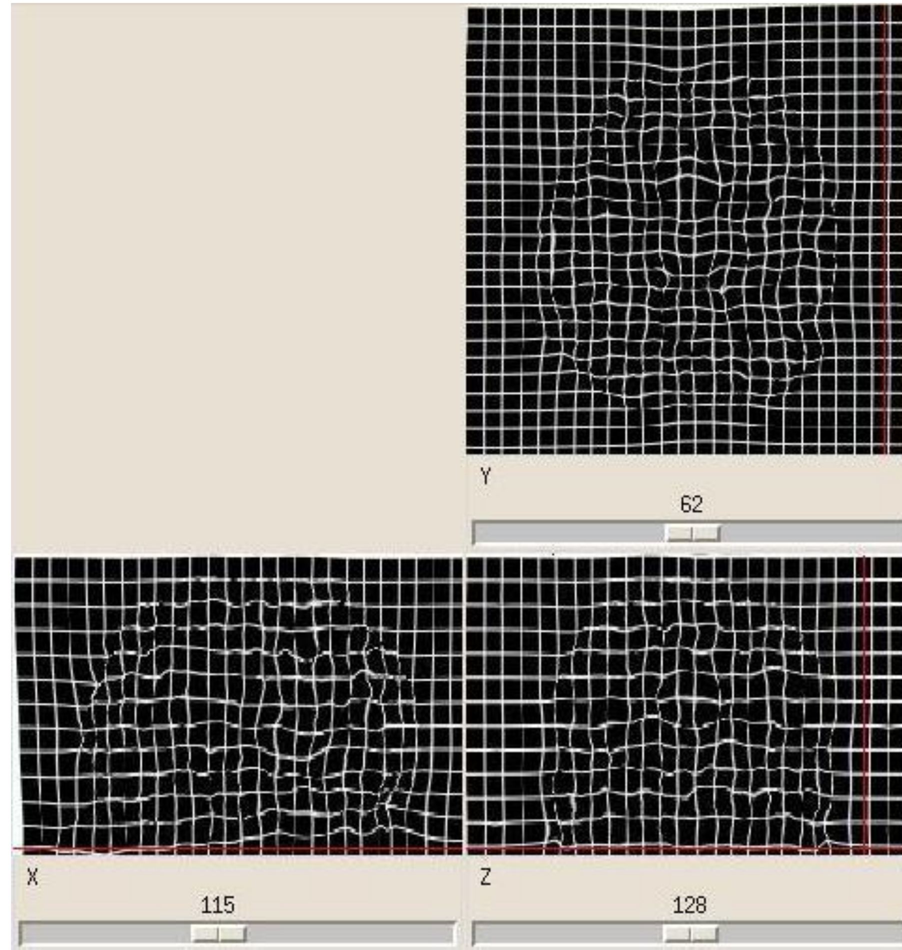
Very good image correspondence

But anatomically meaningless deformation
Jacobian [1/50;50]

Inter-subject registration

Adaptive non-stationary visco-elastic regularization





Registration in 5 min on 15 PCs

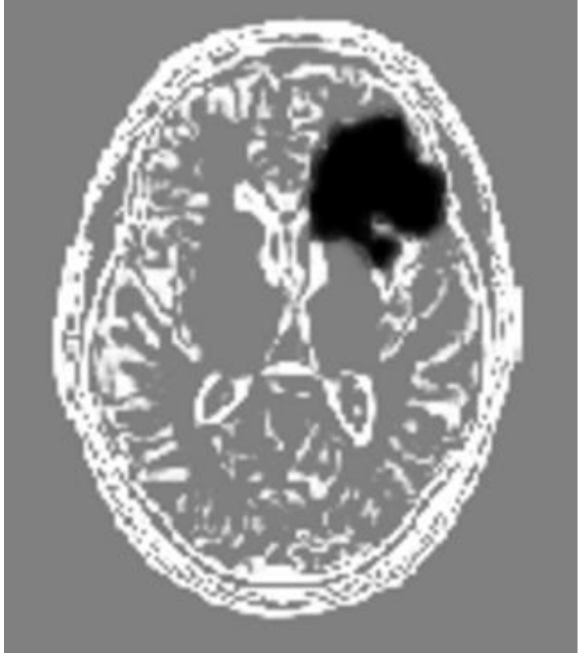
Anatomically more meaningful deformation

Jacobian [1/5;5]

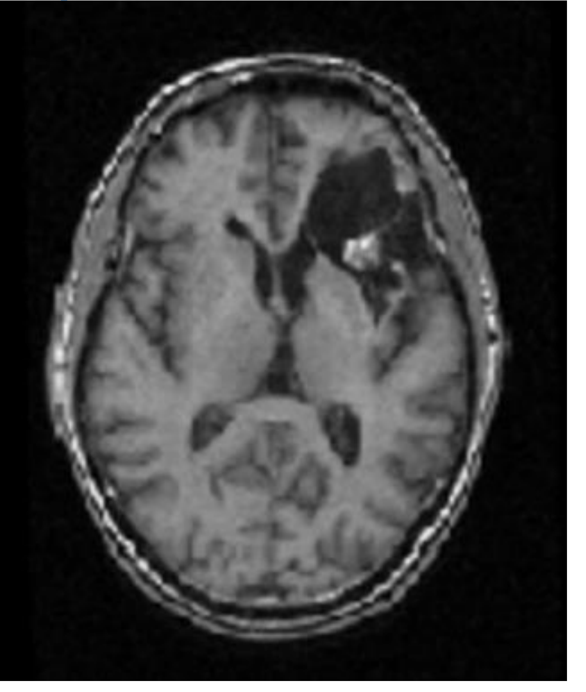
Patient with Pathology

Fuzzy segmentation of the resection

Confidence



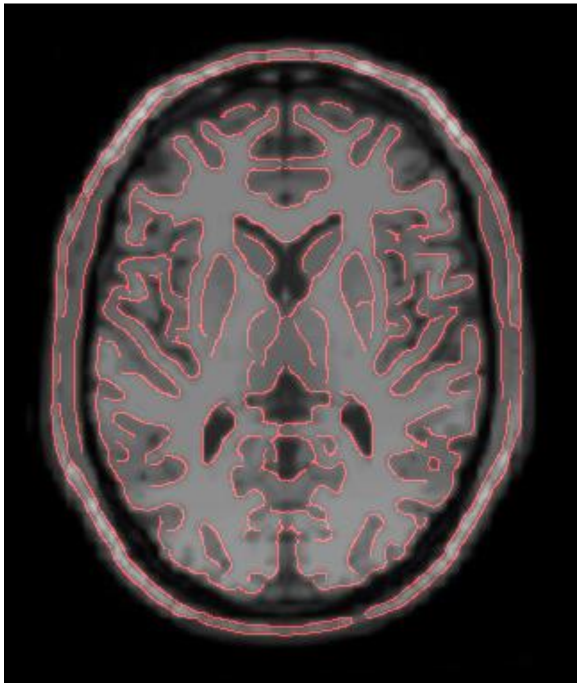
Low confidence values in the resection region

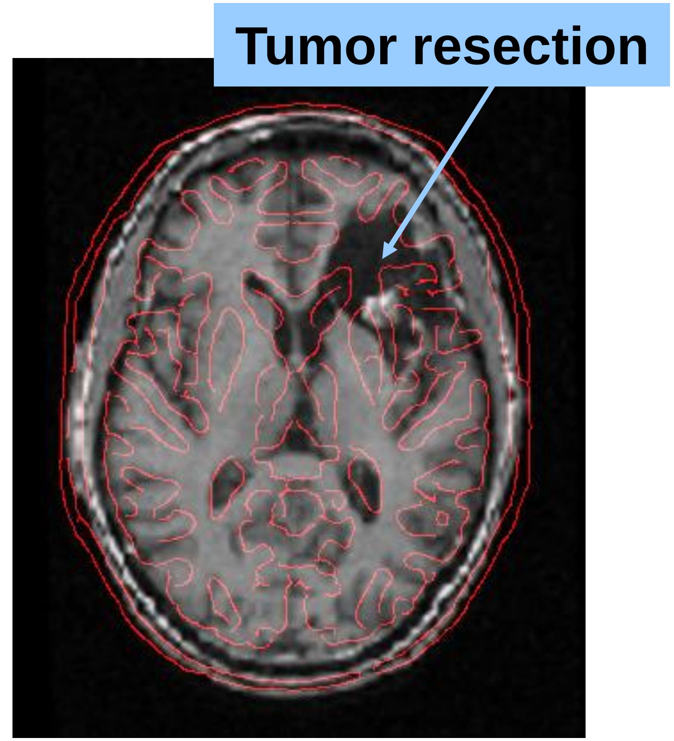


Patient T1-MRI

Atlas and Patient with Pathology

Initialization: affine registration maximizing the correlation ratio





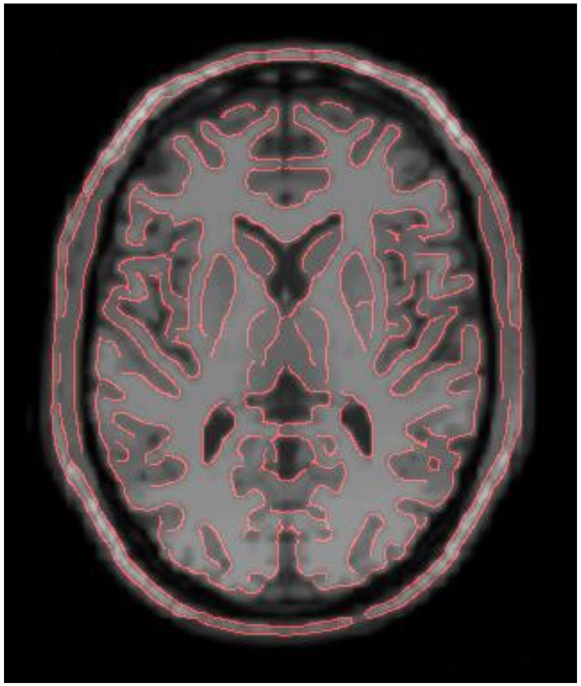
Atlas

Patient T1-MRI

R. Stefanescu, O. Commowick, G. Malandain, P.-Y. Bondiau, N. A., and X. Pennec. Non-Rigid Atlas to Subject Registration with Pathologies for Conformal Brain Radiotherapy. MICCAI'04, 2004.

Data courtesy of Dr. Pierre-Yves Bondiau, M.D., Centre Antoine Lacassagne, Nice, France

Registration Result

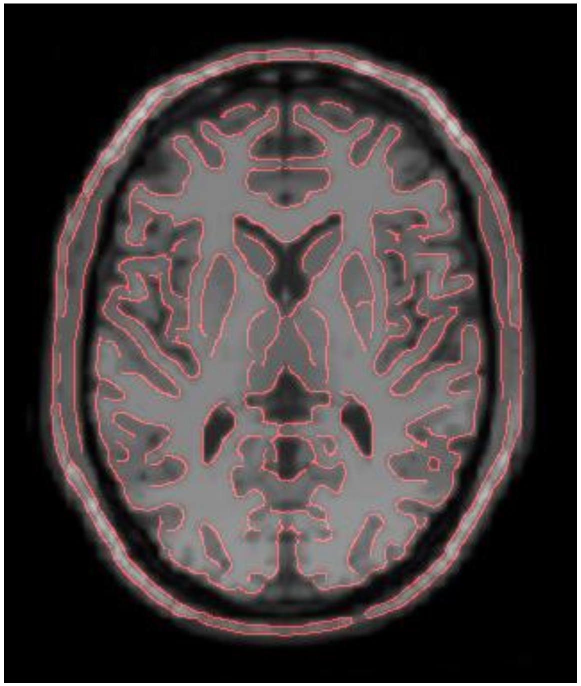


Atlas

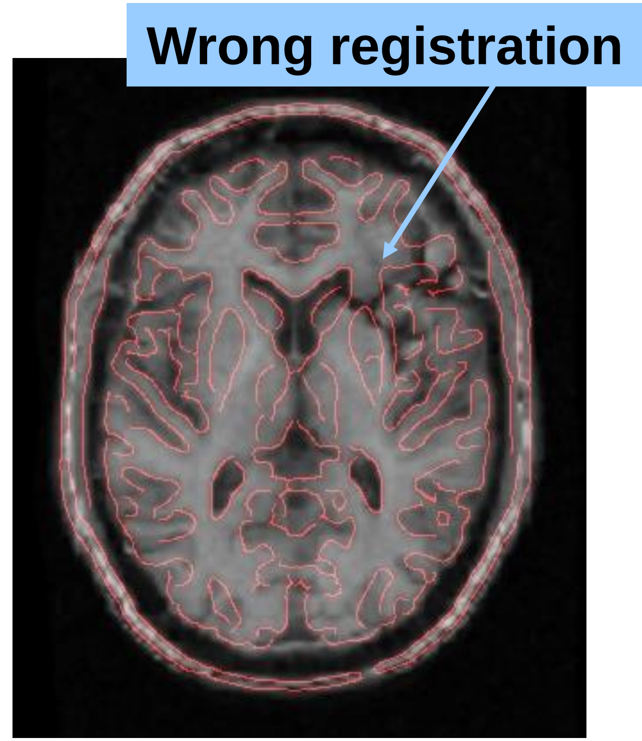
Patient T1-MRI

Data courtesy of Dr. Pierre-Yves Bondiau, M.D., Centre Antoine Lacassagne, Nice, France

Classical (wrong) Registration



Atlas



Patient T1-MRI

Data courtesy of Dr. Pierre-Yves Bondiau, M.D., Centre Antoine Lacassagne, Nice, France

Talk overview

The early phase (Thirion)

A Pair and Smooth approach (Cathier)

Adaptive regularization (Stefanescu)

Diffeomorphic demons (Vercauteren)

Extensions and log-demons (Mansi, Yeo, Vercauteren)

Spatial Transformations Spaces

Most spatial transformation spaces do not form vector spaces but only a Lie group, $\mathcal{G}$

- Rigid-body, projective, diffeomorphisms, etc.

Natural operation: composition

- $\phi_1, \phi_2 \in \mathcal{G} \;\Rightarrow\; \phi = \phi_1 \circ \phi_2 \in \mathcal{G}$, where $\phi(x) = \phi_1(\phi_2(x))$ for $x \in \Omega$

Even if addition exists, often no geometric meaning

- $\phi_1, \phi_2 \in \mathcal{G} \;\Rightarrow\; \phi = \phi_1 + \phi_2 \notin \mathcal{G}$

Many registration algorithms ignore this

Riemannian Metrics on diffeomorphisms

Space of deformations

- Transformation y=ϕ(x)
- Curves in transformation spaces: ϕ(x,t)
- Tangent vector = speed vector field

$$v_t(x) = \frac{d\phi(x,t)}{dt}$$

Right invariant metric

$$\|v_t\|_{\phi_t} = \|v_t \circ \phi_t^{-1}\|_{Id}$$

- Eulerian scheme
- Sobolev Norm H_k or H_∞ (RKHS) in LDDMM → diffeomorphisms [Miller, Trouve, Younes, Holm, Dupuis, Beg… 1998 – 2009]

Geodesics determined by optimization of a time-varying vector field

- Distance

$$d^2(\phi_0, \phi_1) = \arg\min_{v_t} \left(\int_0^1 \|v_t\|_{\phi_t}^2 dt \right)$$

- Geodesics characterized by initial momentum
- Initial momentum can be parameterized finite dimensional parameters

Demons vs LDDMM

Use a smoothing metric on the tangent space

- Gaussian smoothing of update (~ fluid regularization)
- Registration = transformation trajectory in some space

But optimize a different regularizer

- LDDMM regularization = trajectory energy
  - optimize the complete trajectory
- Demons regularization = “elastic” potential
  - optimize the end-point (gradient descent)

Use group properties?

- Right invariant geodesics (LDDMM)
- One-parameter subgroups

The SVF framework for Diffeomorphisms

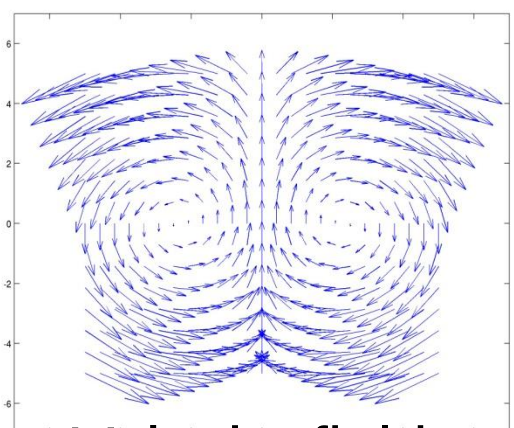
Arsigny et al., MICCAI 06

- Use one-parameter subgroups

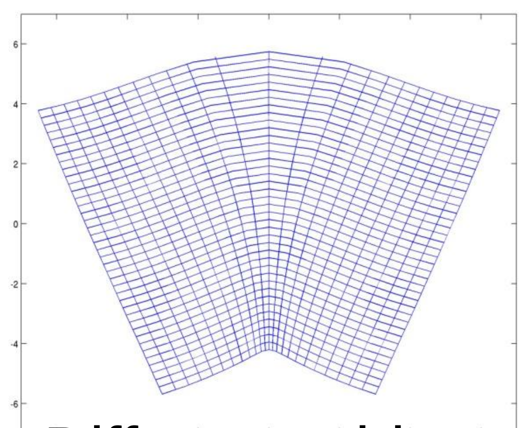
Exponential of a smooth vector field $\boldsymbol{u}$ is a diffeomorphism

- $\boldsymbol{u}$ is a smooth velocity field
- Exponential: solution at time 1 of ODE

$$\partial x(t) / \partial t = \boldsymbol{u}(x(t))$$



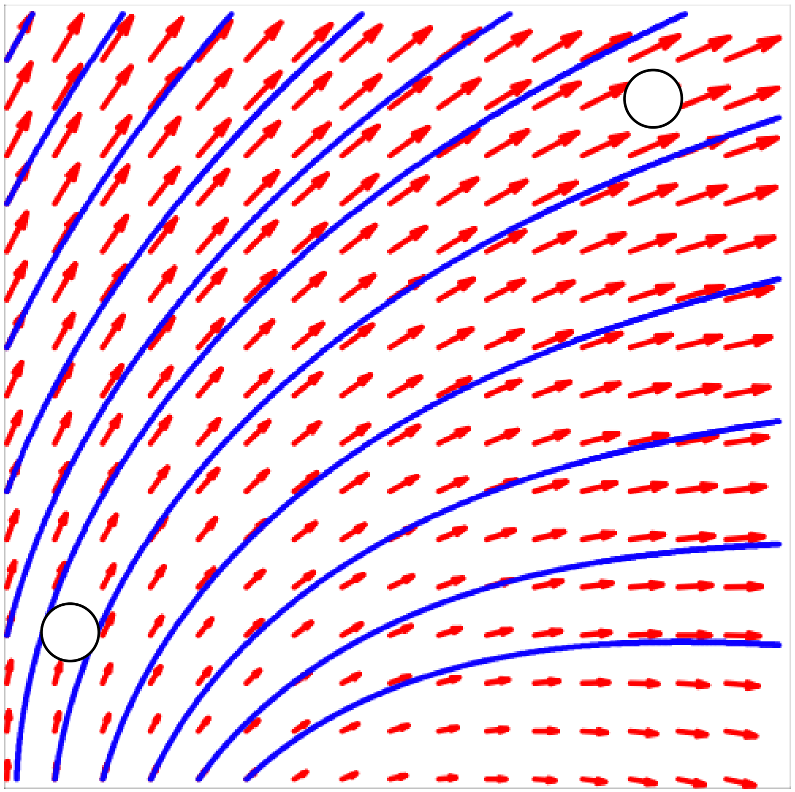
Velocity field



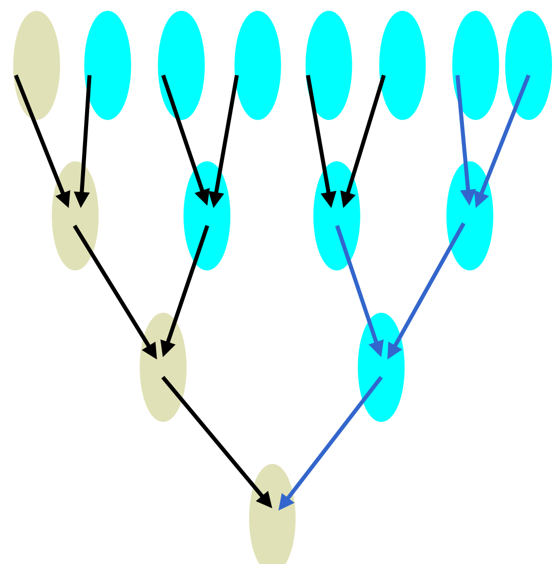
Diffeomorphism

Computing the exponential

$$\exp(\boldsymbol{u}) = \exp(\boldsymbol{u}/N)^N$$



$$\frac{\partial x}{\partial t} = v(x)$$
$$x(0) = x_0$$
$$x(1) = \int_0^1 v(x(t))dt$$
$$\triangleq \exp(v)$$



$\exp(\boldsymbol{v}/8) \approx \mathrm{Id} + \boldsymbol{v}/8$

$\exp(\boldsymbol{v}/4) = \exp(\boldsymbol{v}/8)^2$

$\exp(\boldsymbol{v}/2) = \exp(\boldsymbol{v}/4)^2$

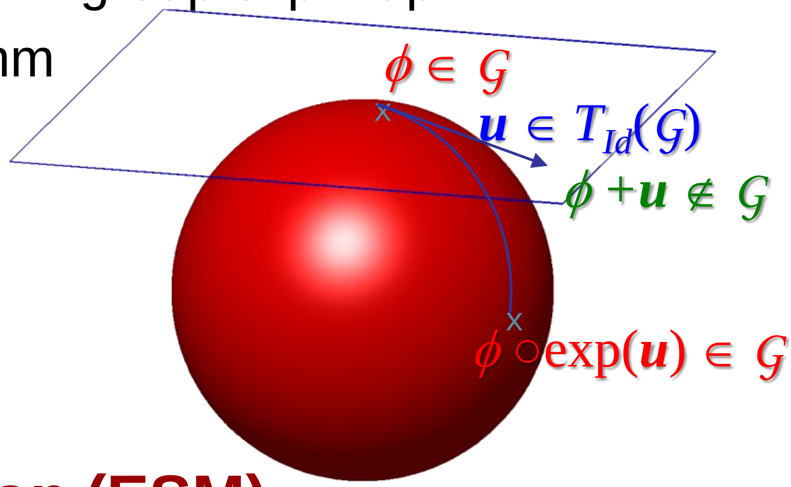
$\exp(\boldsymbol{v})$

•V. Arsigny, O. Commowick, X. Pennec, N. Ayache. A Log-Euclidean Framework for Statistics on Diffeomorphisms. In Proc. of MICCAI'06, LNCS 4190, pages 924-931, 2-4 October 2006.

Diffeomorphic demons

Use Lie group structure on diffeomorphisms to update

- Large deformations by composition with group exp map
- Efficient scaling and squaring algorithm



$$\phi(x) \leftarrow \phi(x) \circ \exp(u)$$

Efficient Second Order Minimization (ESM)

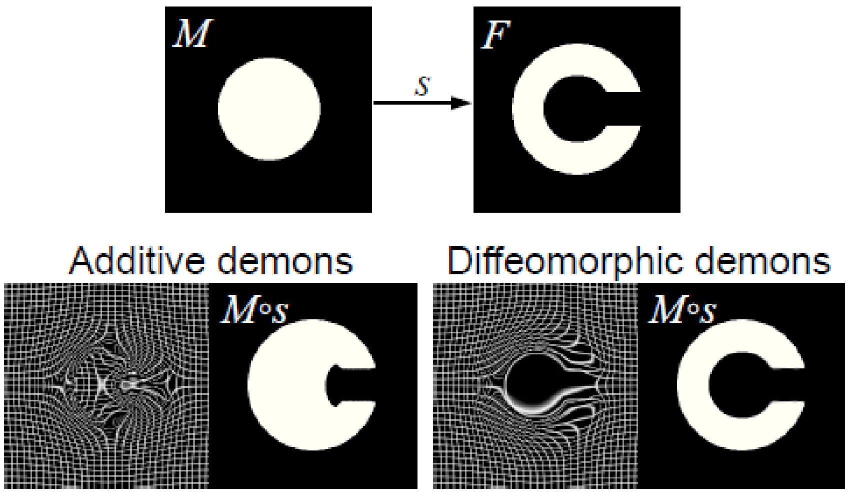
- Error *err(x) = (I-Joϕ)*
- Use first derivatives at 2 points to build 2nd order approx

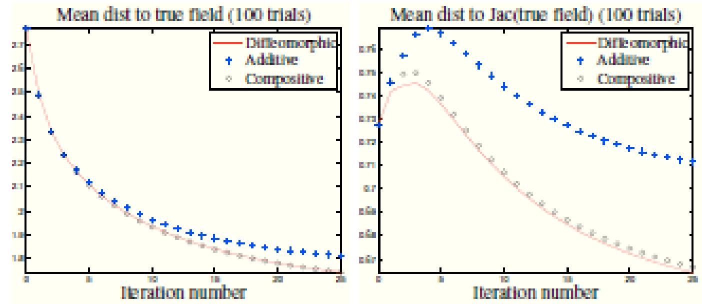
$$\nabla err = -\nabla(J \circ \phi) \text{ (Gauss-Newton)} \rightarrow \nabla err = -\tfrac{1}{2}\left(\nabla I + \nabla(J \circ \phi)\right) \text{ (ESM)}$$

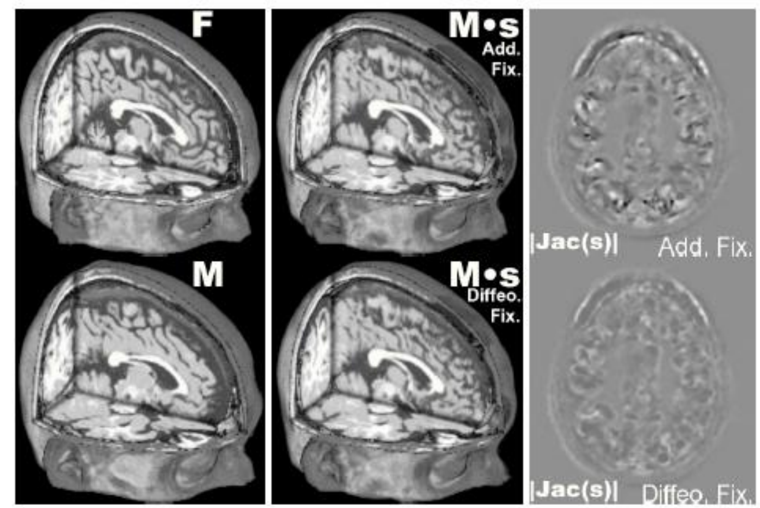
- Solve: $(\nabla err.\nabla err^T + \alpha.Id).u = -err.\nabla err$

[Vercauteren et al Neuroimage 45:(supp 1) S61-72, 2009]

Diffeomorphic demons





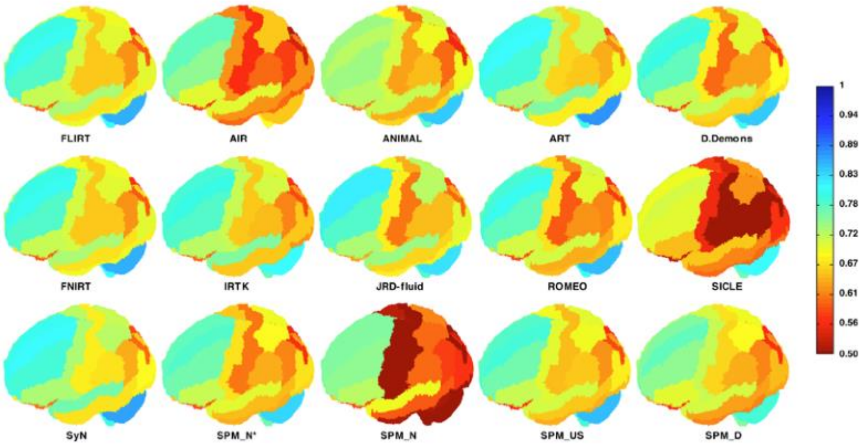


Results

- Really large deformations
- Smoother and non-negative Jacobians
- Faster convergence

[Vercauteren et al Neuroimage 45:(supp 1) S61-72, 2009]
(Open) source-code available at http://hdl.handle.net/1926/510

Large scale evaluation



Klein et al., NeuroImage 09

- 16 groups involved: *MKT, INRIA, LONI, Imperial College, UPenn, UIowa, FMRIB, Wellcome Trust,…*
- 14 registration softwares
- 80 manually segmented brains
- Over 45,000 pairwise registrations performed
- 8 different comparison measures: Dice
- 3 independent statistical tests
- Diffeomorphic Demons : mean rank 3, very fast

Arno Klein, J Andersson, B A. Ardekani, J Ashburner, B Avants, MC Chiang, G E. Christensen, D. L Collins, P Hellier, J H Song, M Jenkinson, C Lepage, D Rueckert, P Thompson, **Tom Vercauteren**, R P. Woods, J. J Mann, and R V. Parsey. *Evaluation of 14 nonlinear deformation algorithms applied to human brain MRI registration.* **NeuroImage,** 2009.

Average Rank

Algorithm	mean rank	dof	run time: minutes	year
SyN	1.00	28M	77 (15.1)	2008
ART	1.00	7M	20.1 (1.6) [Linux]	2005
IRTK	1.63	1.4M	120.8 (29.3)	1999
SPM5 DARTEL Toolbox	1.88	6.4M	71.8 (6.3)	2007
JRD-fluid	2.50	2M	17.1 (1.0) [Solaris]	2007
Diffeomorphic Demons	3.00	21M	8.7 (1.2)	2007
FNIRT	3.00	30K	29.1 (6.0)	2008
ROMEO	3.50	2M	7.5 (0.5)	2001
ANIMAL		69K	11.2 (0.4)	1994
SICLE		8K	33.5 (6.6)	1999
SPM5 Unified Segmentation		1K	≃1	2005
"SPM2-type" Normalize		1K	≃1	1999
SPM5 Normalize		1K	≃1	1999
AIR		168	6.7 (1.5)	1998

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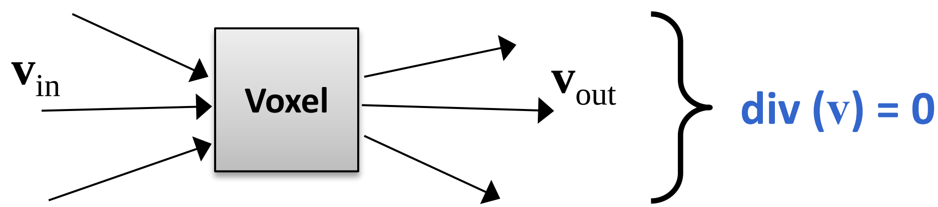
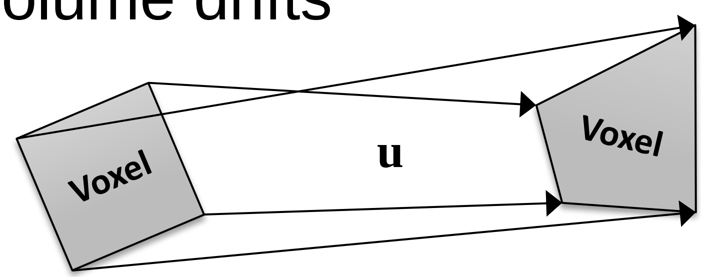
Incompressible demons

In the myocardium, incompressiblity ensured:

1. On the velocities (Eulerian frame): mass continuity equation *(Saddi et al., SPIE, 2008)*



2. On the deformation (Lagrangian frame): correct remaining volume drifts

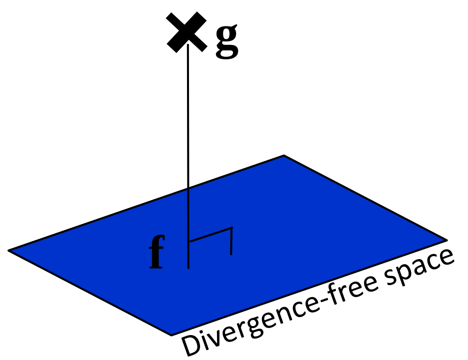


Hard constraint |Jac (**u**) |= 1

(Rohlinf et al, TMI, 2003)

Incompressible demons

- Constraint on update field: div(u) = 0
- Projection onto the space of divergence-free vector fields



$$\mathbf{u} = \Pi(\mathbf{g}) = \mathbf{g} - \mathbf{grad}(p)$$

p solution of:

$$\begin{cases} \Delta p = \mathrm{div}(\mathbf{v}) \\ p = 0 \text{ at the domain boundaries} \end{cases}$$

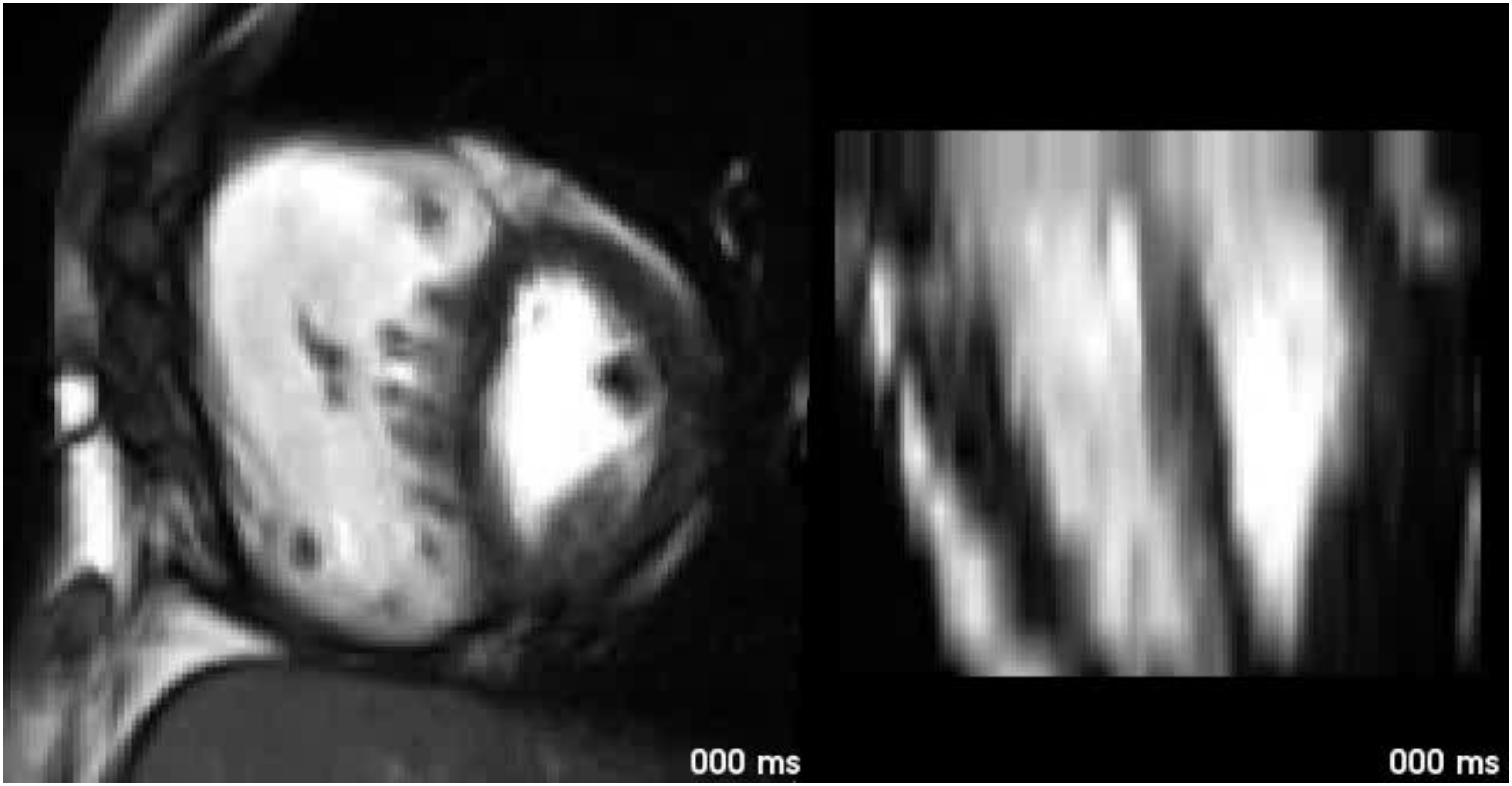
Solve a linear system

- Sparse and constant stiffness matrix
- Limited domain (only myocardium)

➔ ***no significant overhead after preconditionning***

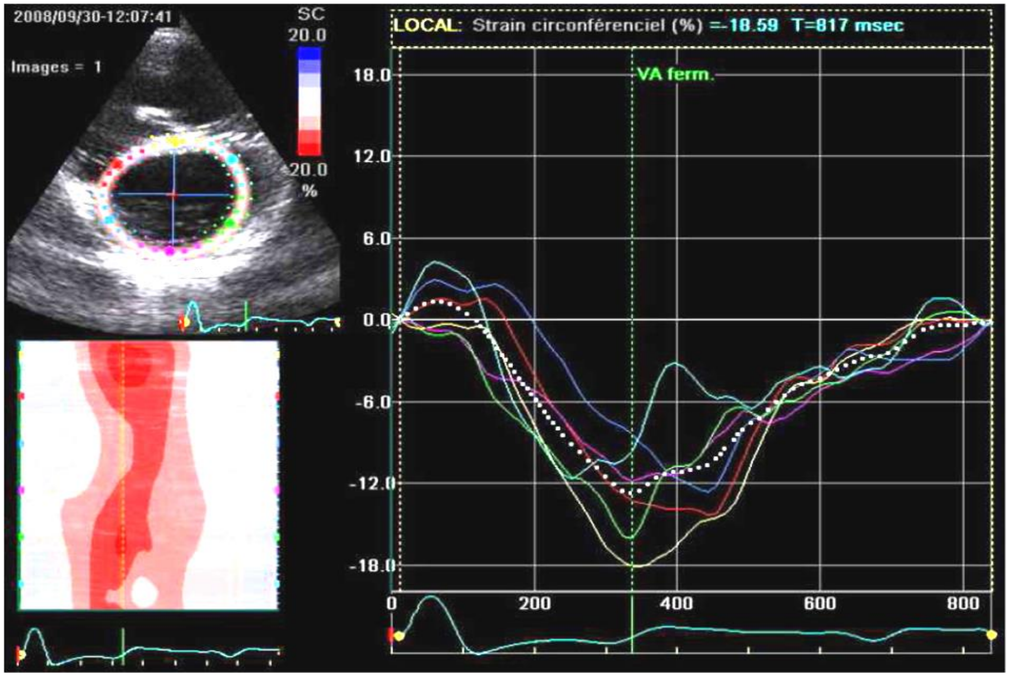
T Mansi, JM Peyrat, M Sermesant, H Delingette, J Blanc, Y Boudjemline, and N Ayache. *Physically-Constrained Diffeomorphic Demons for the Estimation of 3D Myocardium Strain from Cine-MRI*. FIMH 2009

Clinical Evaluation
Patient with Repaired Tetralogy of Fallot

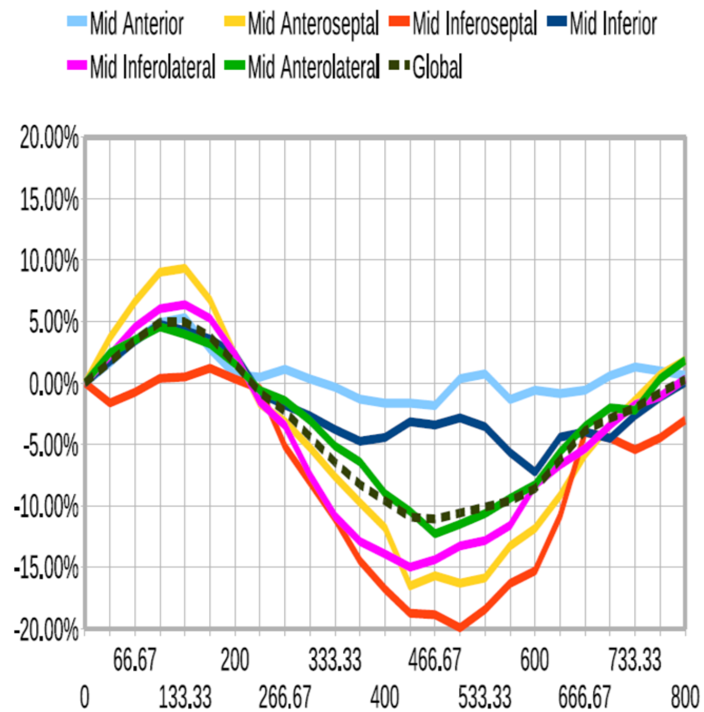


Patient with repaired Tetralogy of Fallot
Circumferential Strain

Patient from Necker – Enfants Malades, Paris



Circumferential strain measured using ultrasound Automatic Function Imaging (GE)



Circumferential strain estimated from short axis cine MRI

➔ *Realistic circumferential strains in ToF*

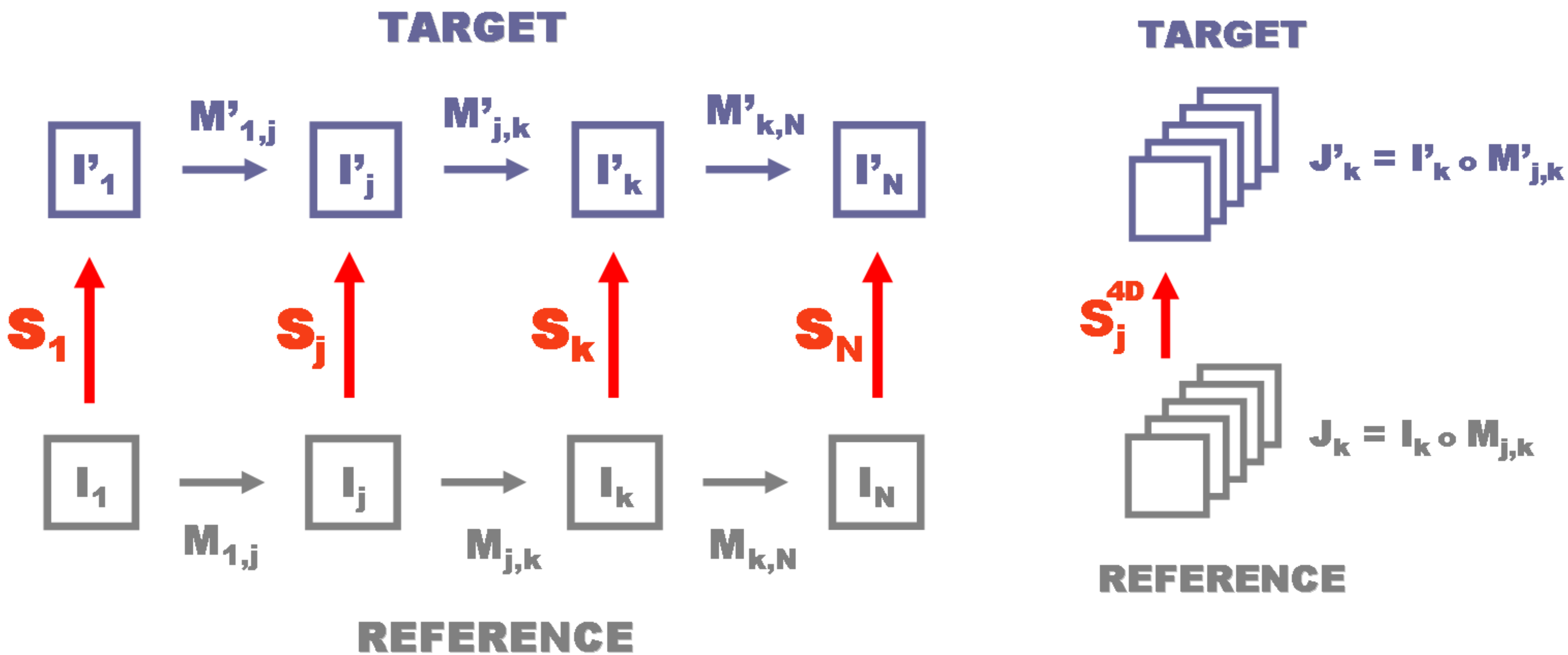
≠ *2D strain in echo: Full 3D - No rater variability!*

Mansi et al., MICCAI 2010; Mansi et al., FIMH 2009

4-D Demons for Cardiac Imaging

Incorporate trajectory constraints:

From 4D to Multichannel Registration



JM Peyrat, H Delingette, M Sermesant, X Pennec, CY Xu, and N Ayache. *Registration of 4D Time-Series of Cardiac Images with Multichannel Diffeomorphic Demons*. MICCAI 2008,

Computing the Update Step

Vector error measure at each voxel (one for each channel):

- $err_i(\phi) = (I_i - J_i o \phi)$
- Taylor expansion: $err(\phi \circ exp(u)) = err(\phi) + \nabla err(\phi)^t .u + O(||u||^2)$
- Beware: $\nabla err(\phi)$ is now a matrix!

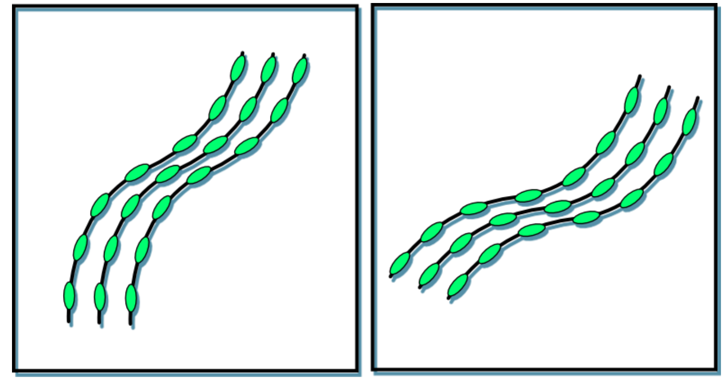
Least squares: Gauss-Newton approximation

$$E(\phi) = \frac{1}{2}\sum_x \left\| err(\phi)(x) \right\|^2 \quad \Rightarrow \quad E(\phi \circ \exp(u)) \approx \frac{1}{2}\sum_x \left\| err(\phi)(x) + \nabla err(\phi)(x)^t .u(x) \right\|^2$$

- Solve $\left(\sum_x \nabla err(\phi)(x).\nabla err(\phi)(x)^t\right).u(x) = \left(\sum_x \nabla err(\phi)(x).err(\phi)(x)\right)$

- Inversion lemma for scalar errors does not work any more:
  Solve a small (dim=num chanels) matrix system at each voxel *x*

DTI registration



Similarity metric:

- Tensor comparison (distance)

$$C(\phi) = \int dist^2\left(\Sigma_1(x), (\phi * \Sigma_2)(x)\right)$$

  - Euclidean, Log-Euclidean….

Deforming tensor images: Tensor re-orientation

- Affine *action* $\phi * \Sigma = D\phi . \Sigma \circ \phi . D\phi^t$ does not preserve eigenvalues [Alexander TMI 20(11) 2001]
- Rotate eigenvectors only: $\phi * \Sigma = R(D\phi) . \Sigma \circ \phi . R(D\phi)^t$
  - Finite-Strain (FS): Closest rotation $R(\phi) = (D\phi . D\phi^t)^{-1/2} D\phi$
    [Zhang et al. MedIA 10(5) 2006 & TMI 26(11) 2007] (locally affine)
  - Preservation of Principal Directions (PPD)
    [Alexander and Gee CVIU 77(2), 2000, Cao et al MMBIA 2006]

***DT-REFinD*: *D*iffusion *T*ensor *R*egistration with *E*xact *Fin*ite-Strain *D*ifferential**

[Yeo, et al. DTI Registration with Exact Finite-Strain Differential. ISBI'08, TMI 2009]

Tensor interpolation/metric

- Euclidean and Log-Euclidean (Arsigny '06)

Tensor reorientation

- Finite Strain: $R(\phi) = (D\phi . D\phi^t)^{-1/2} D\phi$

Exact differential

- How a change in $D\phi$ affect R?
- Solution from Pose estimation [Dorst PAMI 27(2) 2005]

$$dR = -R\,[\,R^T(\,tr((D\phi.D\phi^T)^{1/2})I - (D\phi.D\phi^T)^{1/2}\,)^{-1} \sum (R^T)_i \otimes (d(D\phi)^T)_i\,]^{\oplus}$$

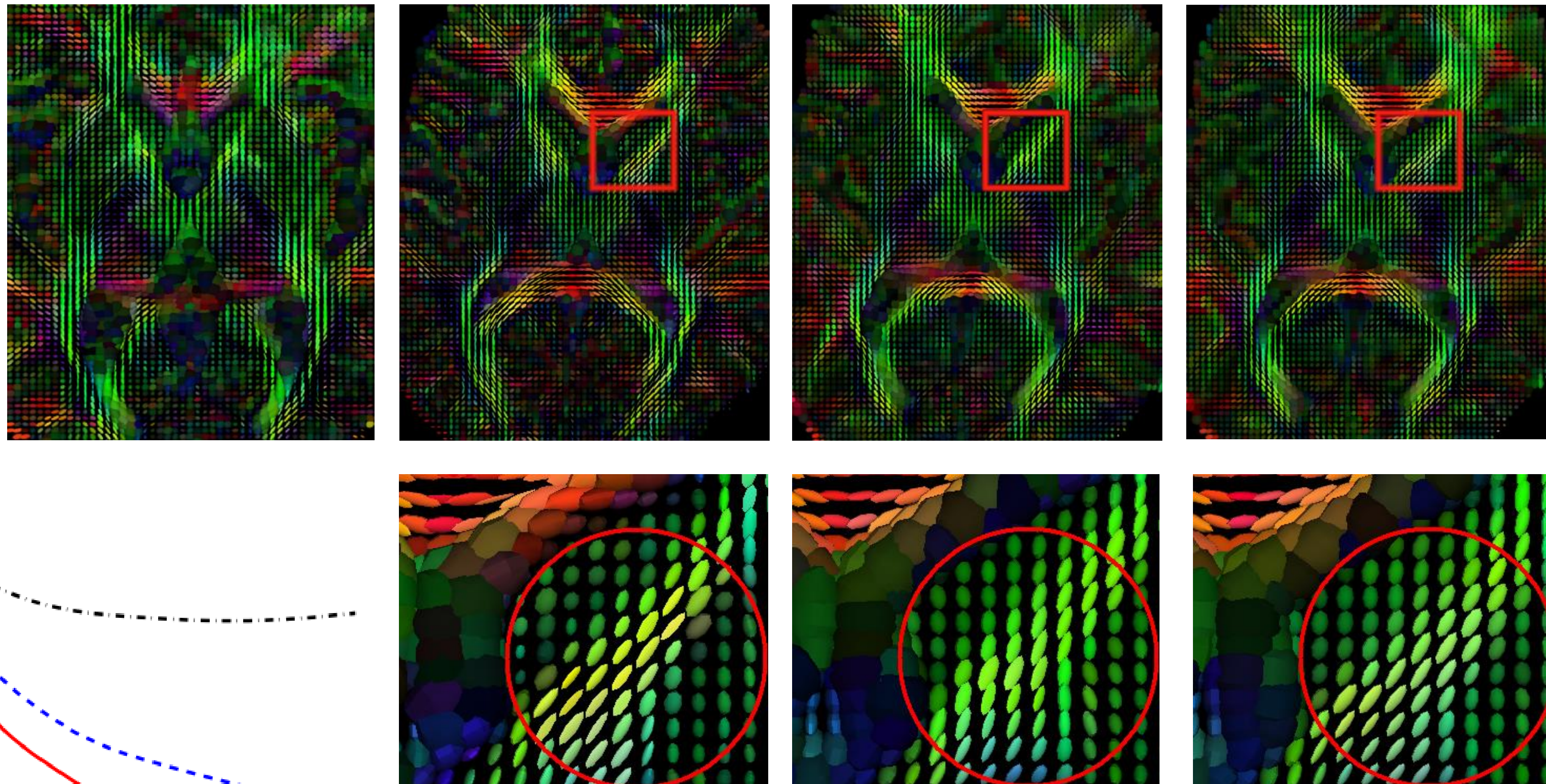
- System to solve for Gauss-Newton is now large because of ($D\phi$) ☹

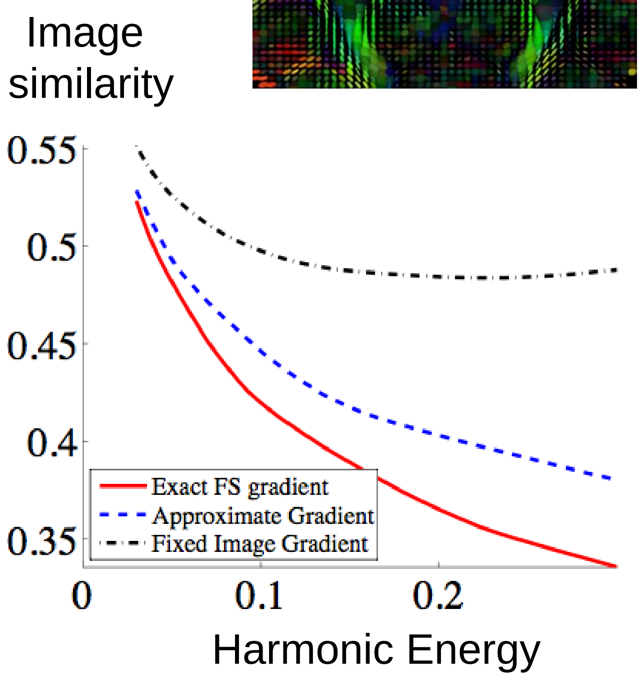
Accurate and still fast

- 15 minutes,128x128x60, Xeon 3.2GHz
- Better tensor alignment

DT-REFinD: Diffusion Tensor Registration with Exact Finite-Strain Differential

Moving Image *M* | Target Image *F* | Approx. Grad (dR=0) | Exact Gradient





15 minutes,128x128x60, Xeon 3.2GHz

T Yeo, T Vercauteren, P Fillard, JM Peyrat, X Pennec, P Golland, N Ayache, and O Clatz, DT-REFinD: *Diffusion Tensor Registration with Exact Finite-Strain Differential*. IEEE Transactions on Medical Imaging, 2009.

Symmetric Log-Demons

Idea: [Arsigny MICCAI 2006, Bossa MICCAI 2007, DARTEL]

- Parameterize the deformation by its logarithm
- Time varying (LDDMM) replaced by stationary vector fields
- Efficient scaling and squaring methods to integrate autonomous ODEs

Parameterize deformation by its Log:

- Replace $s \leftarrow s \circ \exp(\boldsymbol{u})$ by $\exp(\boldsymbol{v}) \leftarrow \exp(\boldsymbol{v}) \circ \exp(\boldsymbol{u})$

$$\mathcal{E}(\mathbf{v}, \mathbf{v}_c) = \underbrace{\frac{1}{\sigma_i^2} \|F - M \circ \exp(\mathbf{v}_c)\|_{L_2}^2}_{\text{Similarity}} + \underbrace{\frac{1}{\sigma_x^2} \|\log(\exp(-\mathbf{v}) \circ \exp(\mathbf{v}_c))\|_{L_2}^2}_{\text{Coupling}} + \underbrace{\mathcal{R}(\mathbf{v})}_{\text{Regularisation}}$$

Similarity: *Measures how much the two images differ*

Coupling: *Couples the correspondences with the smooth deformation*

Regularisation: *Ensures deformation smoothness*

Approximation with BCH formula [Bossa 2007]

- $\exp(\boldsymbol{v}) \circ \exp(\varepsilon\boldsymbol{u}) = \exp(\ \boldsymbol{v} + \varepsilon\boldsymbol{u} + [\boldsymbol{v},\varepsilon\boldsymbol{u}]/2 + [\boldsymbol{v},[\boldsymbol{v},\varepsilon\boldsymbol{u}]]/12 + \dots\ )$
  - Lie bracket $[\boldsymbol{v},\boldsymbol{u}](p) = \mathrm{Jac}(\boldsymbol{v})(p).\boldsymbol{u}(p) - \mathrm{Jac}(\boldsymbol{u})(p).\boldsymbol{v}(p)$

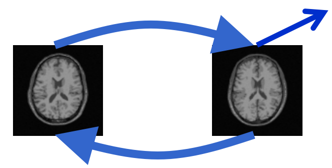
T Vercauteren, X Pennec, A Perchant, and N Ayache. *Symmetric Log-Domain Diffeomorphic Registration: A Demons-based Approach*, MICCAI 2008

Symmetric Log-Demons

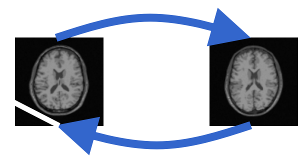
Use easy inverse: $s^{-1} = \exp(-v)$

Iteration

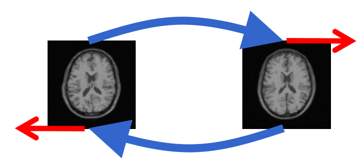
- Given images I_0, I_1 and current transformation $s=\exp(\boldsymbol{v})$
- Forward demons forces $\boldsymbol{u}^{\mathrm{forw}}$



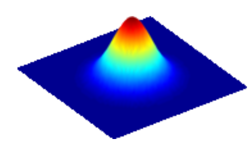
- Backward demons forces $\boldsymbol{u}^{\mathrm{back}}$



- Update
  - $\boldsymbol{v} \leftarrow \frac{1}{2}\left(\mathrm{BCH}(\boldsymbol{v},\boldsymbol{u}^{\mathrm{forw}}) - \mathrm{BCH}(-\boldsymbol{v},\boldsymbol{u}^{\mathrm{back}})\right)$



- Regularize (Gaussian)
  - $\boldsymbol{v} \leftarrow K_{\mathrm{diff}} * \boldsymbol{vc}$



T Vercauteren, X Pennec, A Perchant, and N Ayache. *Symmetric Log-Domain Diffeomorphic Registration: A Demons-based Approach*, MICCAI 2008

Symmetric LCC log-demons

Revised Symmetric LCC-Demons (based on [Cachier 2004])

$$E(\mathbf{v}, \mathbf{v_x}, I, J) = \frac{1}{\sigma_i^2}\rho^2(I, J, \mathbf{v_x}) + \frac{1}{\sigma_x^2}\|\log(\exp(-\mathbf{v}) \circ \exp(\mathbf{v_x}))\|_{L_2}^2 + \frac{1}{\sigma_T^2}\mathrm{Reg}(\mathbf{v})$$

Similarity term (LCC)

$$\rho(I, J, \mathbf{v_x}) = \frac{\mathbf{G}_\sigma * (I \circ \exp(-\frac{\mathbf{v_x}}{2}) \cdot J \circ \exp(\frac{\mathbf{v_x}}{2}))}{\sqrt{\mathbf{G}_\sigma * (I \circ \exp(-\frac{\mathbf{v_x}}{2}))^2 \cdot \mathbf{G}_\sigma * (J \circ \exp(\frac{\mathbf{v_x}}{2}))^2}}$$

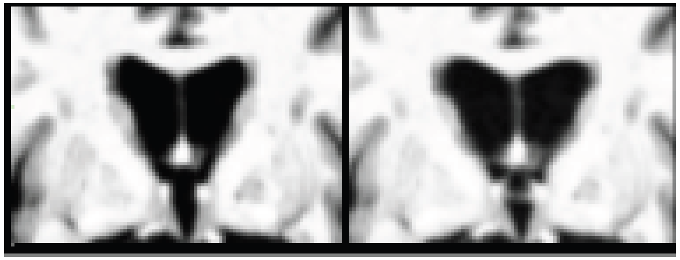
Symmetric similarity

Closed form Demons-like update

(computational efficiency preserved)

$$\delta v = -\frac{2\Lambda}{\|\Lambda\|^2 + \frac{4}{\rho^2}\frac{\sigma_i^2}{\sigma_x^2}}$$

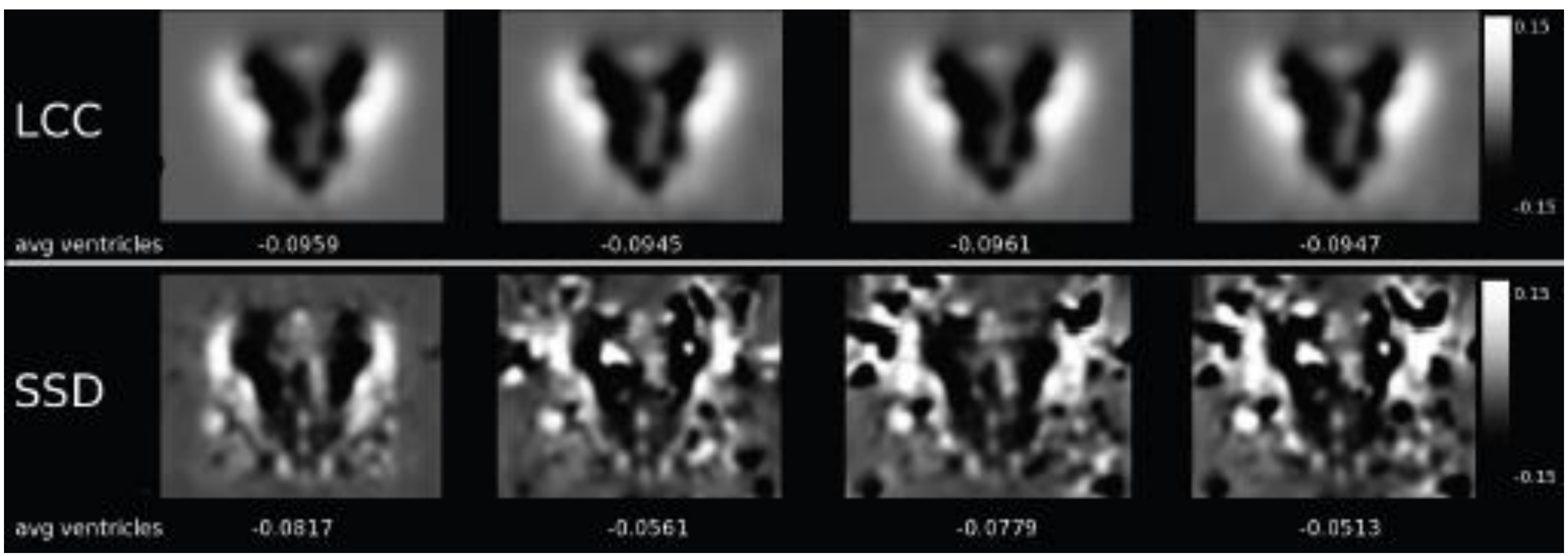
Robustness to the intensity bias



baseline　　synthetic follow-up (ventricles expansion)

Bias: multiplicative　　additive

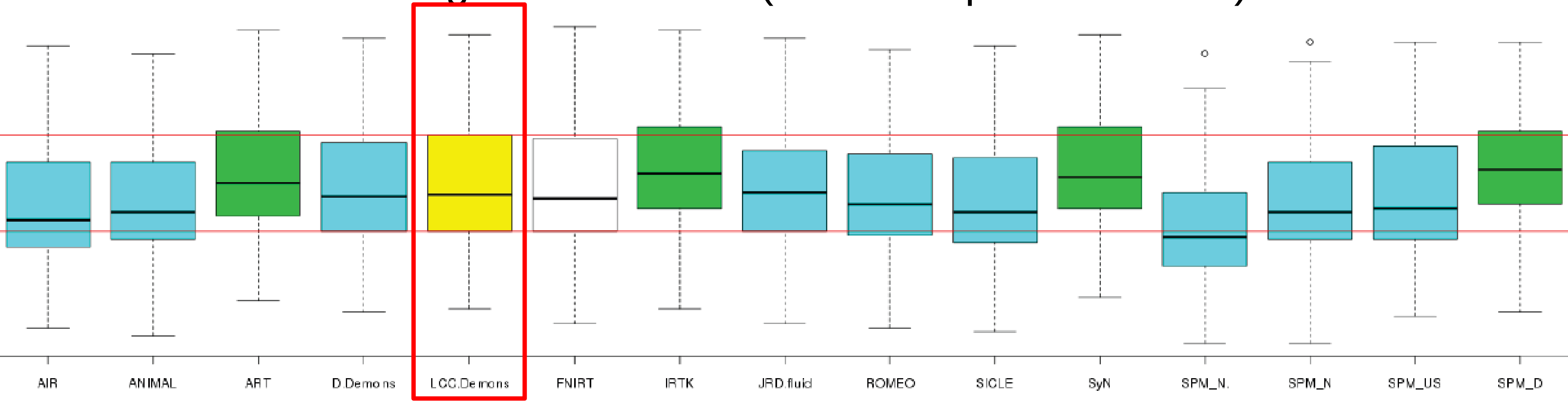
Non-rigid registration LCC-Demons vs standard log-Demons
log-Jacobian determinant of the estimated deformation



No bias　　additive　　multiplicative　　add+mult

Inter-subject registration (Klein study)

Target overlap on 131 manually labeled brain regions
for 144 registrations tests (CUMC12 public dataset)



Significantly higher TO, Significantly lower TO, White boxes: no differences

Intra-subject registration

% whole brain 1 year changes in Alzheimer's disease (AD) (141 AD patients, 200 healthy controls)

Group	% change	
	LCC-Demons	KNBSI
Ctrls	1.09 (1.02)	1.069 (0.925)
AD	1.81 (1.06)	1.714 (0.989)
Sample size (95% CI)	544 (315,1255)	590 (332,1328)

Statistically powered measures of longitudinal brain atrophy

A zoo of demons registration algorithms

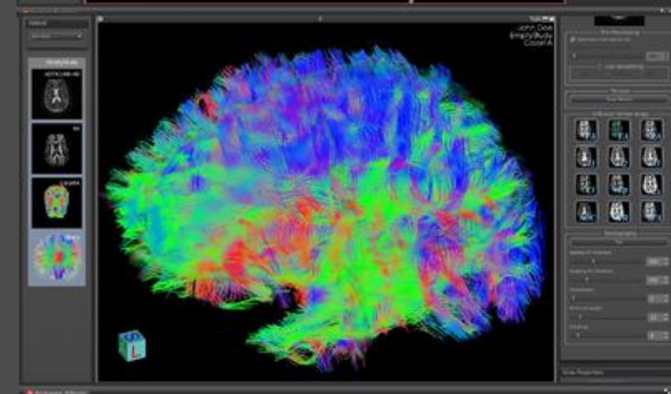
Demons

- Diffeomorphic demons (Vercauteren)
  http://www.insight-journal.org/browse/publication/154
- Spherical demons for inflated brain surfaces (Yeo / Vercauteren)
- Multichannel demons for 4D registration of cardiac sequences (Peyrat)

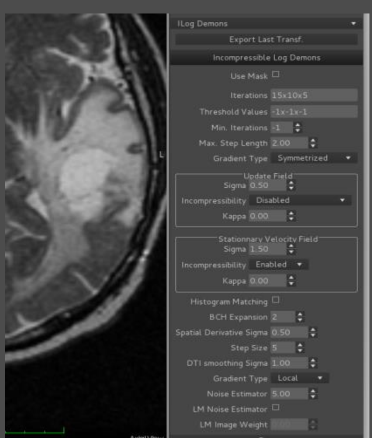
Log Demons

- Open-source ITK implementation (Vercauteren MICCAI 2008)
  http://hdl.handle.net/10380/3060 **[MICCAI Young Scientist Impact award 2013]**
- Matlab version (Hervé Lombaert)
  http://www.mathworks.com/matlabcentral/fileexchange/39194-diffeomorphic-log-demons-image-registration
- LCC time-consistent log-demons for AD is publicly available
  http://team.inria.fr/asclepios/software/lcclogdemons/
- Tensor (DTI) demons (Yeo) and log-demons (Sweet WBIR 2010):
  http://gforge.inria.fr/projects/ttk
- 3D myocardium strain / incompressible deformations using Helmoltz decomposition (Mansi MICCAI'10) **http://med.inria.fr**
- Hierarchical multiscale polyaffine log-demons (Seiler, Media 2012)
  [MICCAI 2011 best paper award] **http://web.stanford.edu/~cseiler/software.html**

medInria



- Medical image processing and visualization software
- Open-source, BSD license
- Extensible via plugins
- Provides high-level algorithms to end-users
- Ergonomic and reactive user interface



Available registration algorithms :

- Diffeomorphic Demons
- Incompressible Log Demons
- LCC Log Demons in next release (April 2014)

http://med.inria.fr

INRIA teams involved: Asclepios, Athena, Parietal, Visages